Basic Ground-Water Hydrology

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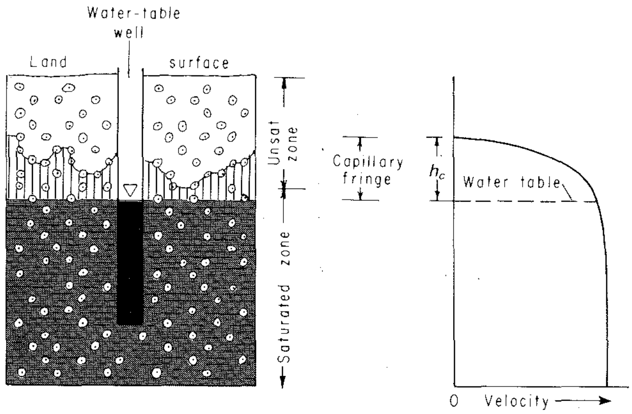
GROUND-WATER VELOCITY



The rate of movement of ground water is important in many problems, particularly those related to pollution. For example, if a harmful substance is introduced into an aquifer upgradient from a supply well, it becomes a matter of great urgency to estimate when the substance will reach the well.

The rate of movement of ground water is greatly overestimated by many people, including those who think in terms of ground water moving through "veins" and underground rivers at the rates commonly observed in surface streams. It would be more appropriate to compare the rate of movement of ground water to the movement of water in the middle of a very large lake being drained by a very small stream.

The *ground-water velocity* equation can be derived from a combination of Darcy's law and the velocity equation of hydraulics.

$$Q=KA\left(\frac{dh}{dl}\right) \qquad \text{(Darcy's law)}$$

$$Q=Av \qquad \text{(velocity equation)}$$

where Q is the rate of flow or volume per unit of time, K is the hydraulic conductivity, A is the cross-sectional area, at a right angle to the flow direction, through which the flow Q occurs, dh/dl is the hydraulic gradient, and v is the Darcian velocity, which is the average velocity of the entire cross-sectional area. Combining these equations, we obtain

$$Av=KA\left(\frac{dh}{dl}\right)$$

Canceling the area terms, we find that

$$v=K\left(\frac{dh}{dl}\right)$$

Because this equation contains terms for hydraulic conductivity and gradient only, it is not yet a complete expression of ground-water velocity. The missing term is porosity (n) because, as we know, water moves only through the openings in a rock. Adding the porosity term, we obtain

$$v=\frac{Kdh}{ndl} \qquad (1)$$

In order to demonstrate the relatively slow rate of ground-water movement, equation 1 is used to determine the rate of movement through an aquifer and a confining bed.

1. *Aquifer composed of coarse sand*

$K=60$ m/d

$dh/dl=1$ m/1,000 m

$n=0.20$

$$v=\frac{K}{n}\times\frac{dh}{dl}=\frac{60\text{ m}}{\text{d}}\times\frac{1}{0.20}\times\frac{1\text{ m}}{1{,}000\text{ m}}$$

$$=\frac{60\text{ m}^2}{200\text{ m d}}=0.3\text{ m d}^{-1}$$

2. *Confining bed composed of clay*

$K=0.0001$ m/d

$dh/dl=1$ m/10 m

$n=0.50$

$$v=\frac{0.0001\text{ m}}{\text{d}}\times\frac{1}{0.50}\times\frac{1\text{ m}}{10\text{ m}}$$

$$=\frac{0.0001\text{ m}^2}{5\text{ m d}}=0.00002\text{ m d}^{-1}$$

Velocities calculated with equation 1 are, at best, average values. Where ground-water pollution is involved, the fastest rates of movement may be several times the average rate. Also, the rates of movement in limestone caverns, lava tubes, and large rock fractures may approach those observed in surface streams.

Further, movement in unconfined aquifers is not limited to the zone below the water table or to the saturated zone. Water in the capillary fringe is subjected to the same hydraulic gradient that exists at the water table; water in the capillary fringe moves, therefore, in the same direction as the ground water.

As the accompanying sketch shows, the rate of lateral movement in the capillary fringe decreases in an upward direction and becomes zero at the top of the fringe. This consideration is important where unconfined aquifers are polluted with gasoline and other substances less dense than water.

TRANSMISSIVITY

The capacity of an aquifer to transmit water of the prevailing kinematic viscosity is referred to as its transmissivity. The *transmissivity* (T) of an aquifer is equal to the hydraulic conductivity of the aquifer multiplied by the saturated thickness of the aquifer. Thus,

$$T = Kb \qquad (1)$$

where T is transmissivity, K is hydraulic conductivity, and b is aquifer thickness.

As is the case with hydraulic conductivity, transmissivity is also defined in terms of a unit hydraulic gradient.

If equation 1 is combined with Darcy's law (see "Hydraulic Conductivity"), the result is an equation that can be used to calculate the quantity of water (q) moving through a unit width (w) of an aquifer. Darcy's law is

$$q = KA\left(\frac{dh}{dl}\right)$$

Expressing area (A) as bw, we obtain

$$q = Kbw\left(\frac{dh}{dl}\right)$$

Next, expressing transmissivity (T) as Kb, we obtain

$$q = Tw\left(\frac{dh}{dl}\right) \qquad (2)$$

Equation 2 modified to determine the quantity of water (Q) moving through a large width (W) of an aquifer is

$$Q = TwW\left(\frac{dh}{dl}\right)$$

or, if it is recognized that T applies to a unit width (w) of an aquifer, this equation can be stated more simply as

$$Q = TW\left(\frac{dh}{dl}\right) \qquad (3)$$

If equation 3 is applied to sketch 1, the quantity of water flowing out of the right-hand side of the sketch can be calculated by using the values shown on the sketch, as follows:

$$T = Kb = \frac{50\text{ m}}{\text{d}} \times \frac{100\text{ m}}{1} = 5{,}000\text{ m}^2\text{ d}^{-1}$$

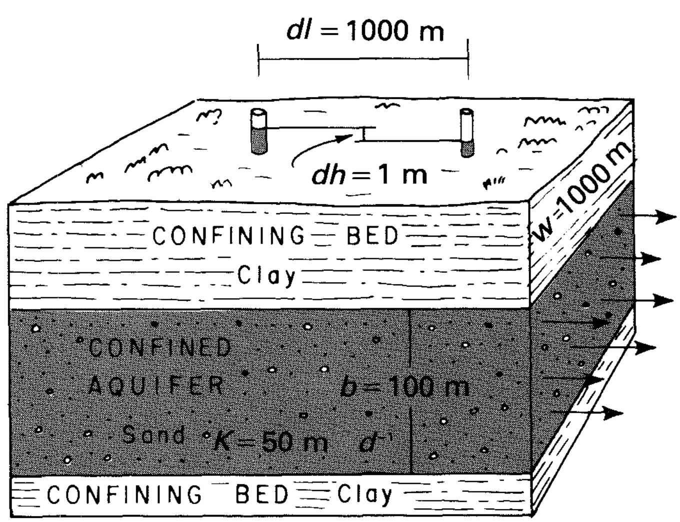
$$Q = TW\left(\frac{dh}{dl}\right) = \frac{5{,}000\text{ m}^2}{\text{d}} \times \frac{1{,}000\text{ m}}{1} \times \frac{1\text{ m}}{1{,}000\text{ m}} = 5{,}000\text{ m}^3\text{ d}^{-1}$$

Equation 3 is also used to calculate transmissivity, where the quantity of water (Q) discharging from a known width of aquifer can be determined as, for example, with streamflow measurements. Rearranging terms, we obtain

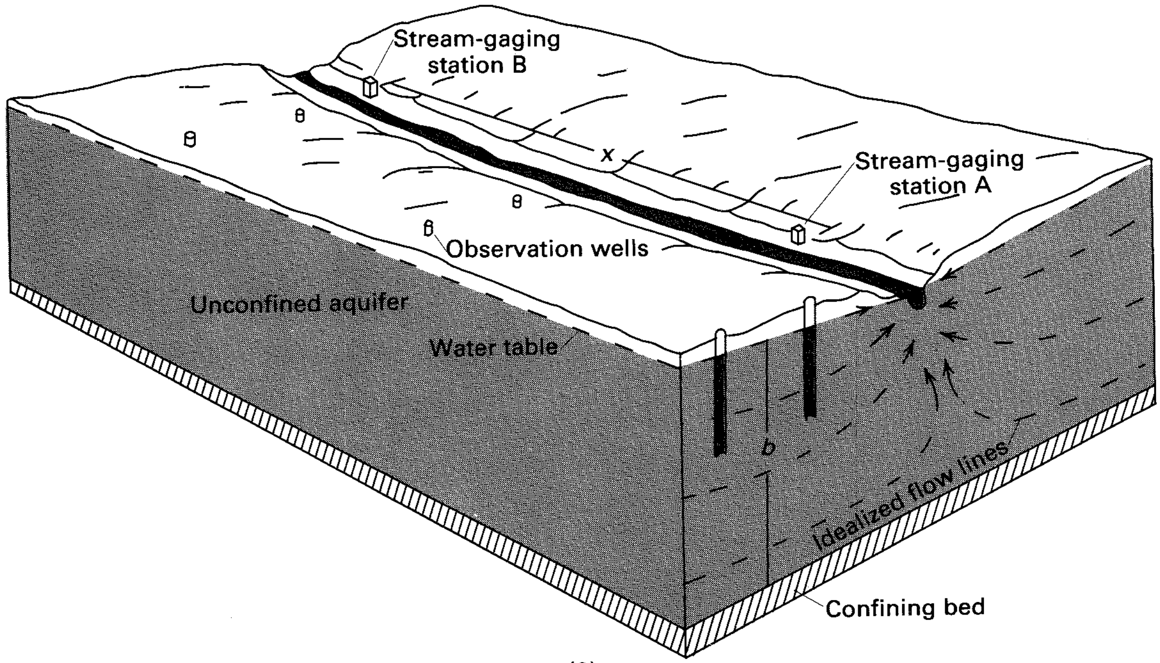
$$T = \frac{Q}{W}\left(\frac{dl}{dh}\right) \qquad (4)$$

The units of transmissivity, as the preceding equation demonstrates, are

$$T = \frac{(\text{m}^3\text{ d}^{-1})(\text{m})}{(\text{m})(\text{m})} = \frac{\text{m}^2}{\text{d}}$$



(1)



(2)

Sketch 2 illustrates the hydrologic situation that permits calculation of transmissivity through the use of stream discharge. The calculation can be made only during dry-weather (baseflow) periods, when all water in the stream is derived from ground-water discharge. For the purpose of this example, the following values are assumed:

Average daily flow at stream-gaging station A:	2.485 $m^3\ s^{-1}$
Average daily flow at stream-gaging station B:	2.355 $m^3\ s^{-1}$
Increase in flow due to ground-water discharge:	0.130 $m^3\ s^{-1}$
Total *daily* ground-water discharge to stream:	11,232 $m^3\ d^{-1}$
Discharge from half of aquifer (one side of the stream):	5,616 $m^3\ d^{-1}$
Distance (x) between stations A and B:	5,000 m
Average thickness of aquifer (b):	50 m
Average slope of the water table (dh/dl) determined from measurements in the observation wells:	1 m/2,000 m

By equation 4,

$$T = \frac{Q}{W} \times \frac{dl}{dh} = \frac{5{,}616\ m^3}{d \times 5{,}000\ m} \times \frac{2{,}000\ m}{1 m} = 2{,}246\ m^2\ d^{-1}$$

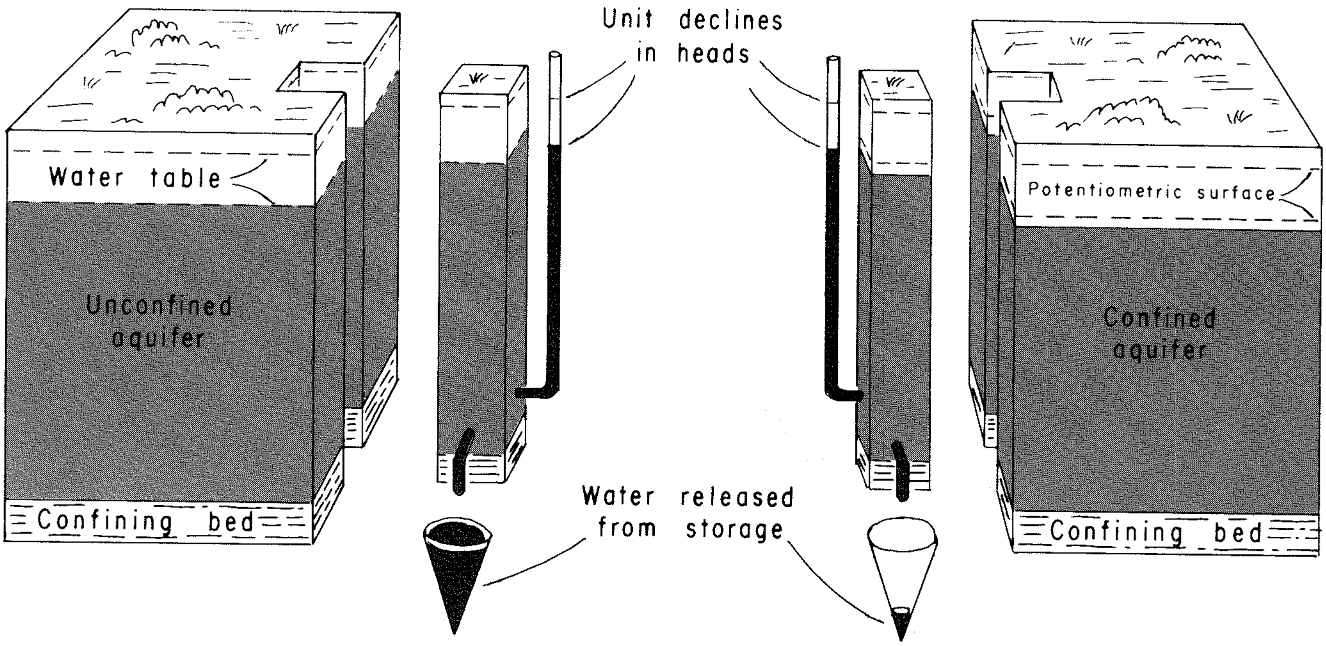
The hydraulic conductivity is determined from equation 1 as follows:

$$K = \frac{T}{b} = \frac{2{,}246\ m^2}{d \times 50\ \ m} = 45\ m\ d^{-1}$$

Because transmissivity depends on both K and b, its value differs in different aquifers and from place to place in the same aquifer. Estimated values of transmissivity for the principal aquifers in different parts of the country range from less than 1 $m^2\ d^{-1}$ for some fractured sedimentary and igneous rocks to 100,000 $m^2\ d^{-1}$ for cavernous limestones and lava flows.

Finally, transmissivity replaces the term "coefficient of transmissibility" because, by convention, an aquifer is transmissive, and the water in it is transmissible.

STORAGE COEFFICIENT



(1)

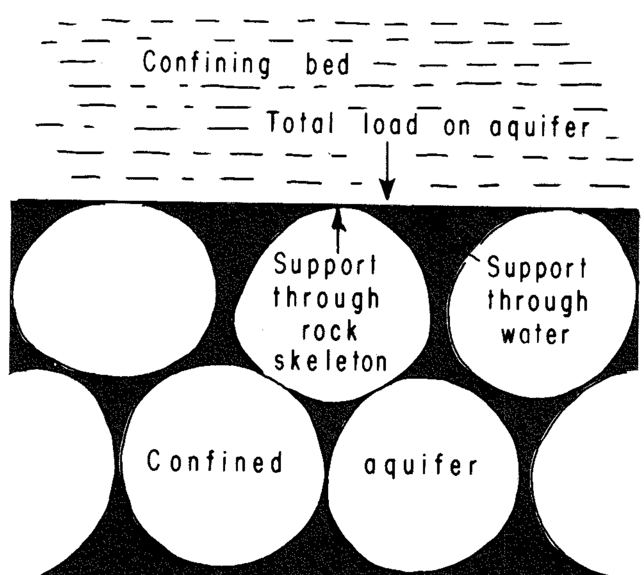
The abilities (capacities) of water-bearing materials to store and to transmit water are their most important hydraulic properties. Depending on the intended use of the information, these properties are given either in terms of a unit cube of the material or in terms of a unit prism of an aquifer.

Property	*Unit cube of material*	*Unit prism of aquifer*
Transmissive capacity	Hydraulic conductivity (K)	Transmissivity (T)
Available storage	Specific yield (S_y)	Storage coefficient (S)

The storage coefficient (S) is defined as the volume of water that an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head. The storage coefficient is a dimensionless unit, as the following equation shows, in which the units in the numerator and the denominator cancel:

$$S = \frac{\text{volume of water}}{\text{(unit area)(unit head change)}} = \frac{(\text{m}^3)}{(\text{m}^2)(\text{m})} = \frac{\text{m}^3}{\text{m}^3}$$

The size of the storage coefficient depends on whether the aquifer is confined or unconfined (1). If the aquifer is confined, the water released from storage when the head declines comes from expansion of the water and from compression of the aquifer. Relative to a confined aquifer, the expansion of a given volume of water in response to a decline in pressure is very small. In a confined aquifer having a porosity of 0.2 and containing water at a temperature of about 15°C, expansion of the water alone releases about 3×10^{-7} m^3 of water per cubic meter of aquifer per meter of decline in head. To determine the storage coefficient of an aquifer due to expansion of the water, it is necessary to multiply the aquifer thickness by 3×10^{-7}. Thus, if only the expansion of water is considered, the storage coefficient of an aquifer 100 m thick would be 3×10^{-5}. The storage coefficient of most confined aquifers ranges from about 10^{-5} to 10^{-3} (0.00001 to 0.001). The difference between these values and the value due to expansion of the water is attributed to compression of the aquifer.



(2)

Sketch 2 will aid in understanding this phenomenon. It shows a microscopic view of the contact between an aquifer and the overlying confining bed. The total load on the top of the aquifer is supported partly by the solid skeleton of the aquifer and partly by the hydraulic pressure exerted by the water in the aquifer. When the water pressure declines, more of the load must be supported by the solid skeleton. As a result, the rock particles are distorted, and the pore space is reduced. The water forced from the pores when their volume is reduced represents the part of the storage coefficient due to compression of the aquifer.

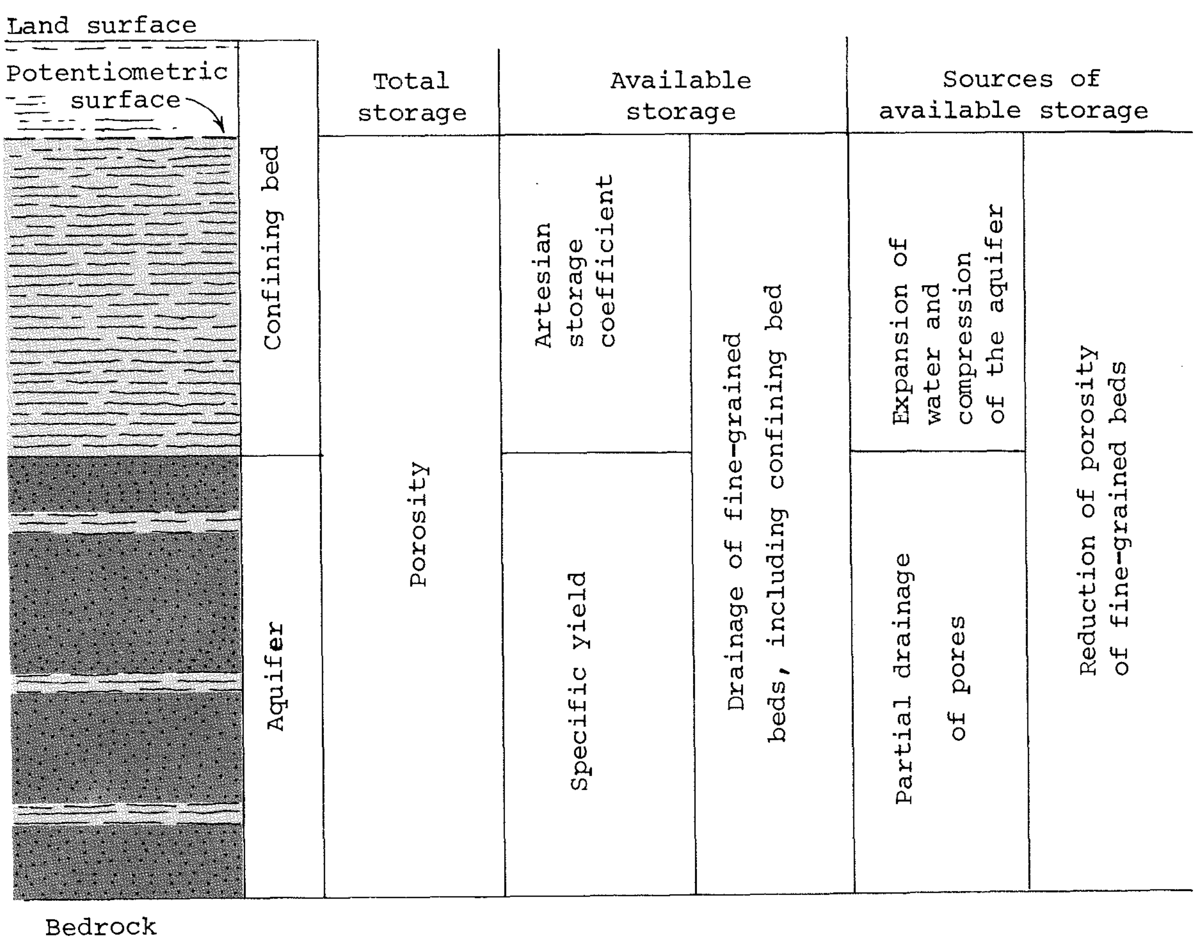
If the aquifer is unconfined, the predominant source of water is from gravity drainage of the sediments through which the decline in the water table occurs. In an unconfined aquifer, the volume of water derived from expansion of the water and compression of the aquifer is negligible. Thus, in such an aquifer, the storage coefficient is virtually equal to the specific yield and ranges from about 0.1 to about 0.3.

Because of the difference in the sources of storage, the storage coefficient of unconfined aquifers is 100 to 10,000 times the storage coefficient of confined aquifers (1). However, if water levels in an area are reduced to the point where an aquifer changes from a confined condition to an unconfined condition, the storage coefficient of the aquifer immediately increases from that of a confined aquifer to that of an unconfined aquifer.

Long-term withdrawals of water from many confined aquifers result in drainage of water both from clay layers within the aquifer and from adjacent confining beds. This drainage increases the load on the solid skeleton and results in compression of the aquifer and subsidence of the land surface. Subsidence of the land surface caused by drainage of clay layers has occurred in Arizona, California, Texas, and other areas.

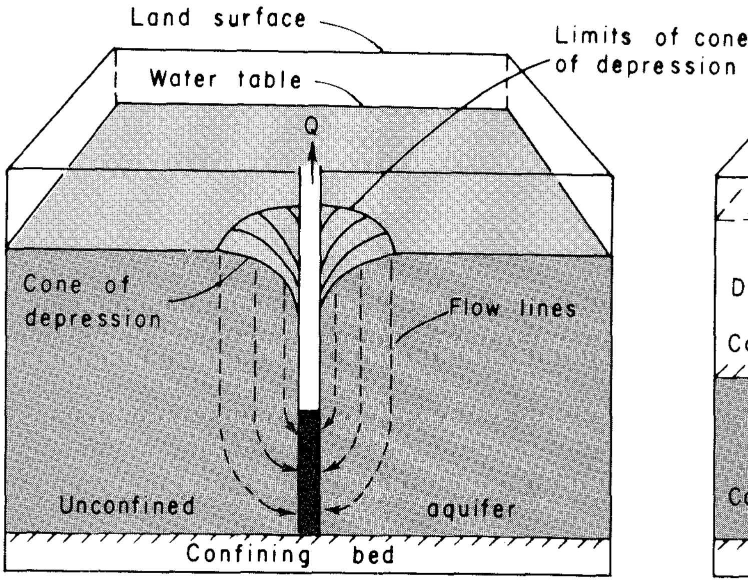
The potential sources of water in a two-unit ground-water system consisting of a confining bed and a confined aquifer are shown in sketch 3. The sketch is based on the assumption that water is removed in two separate stages—the first while the potentiometric surface is lowered to the top of the aquifer and the second by dewatering the aquifer.

The differences in the storage coefficients of confined and unconfined aquifers are of great importance in determining the response of the aquifers to stresses such as withdrawals through wells. (See "Well-Field Design.")

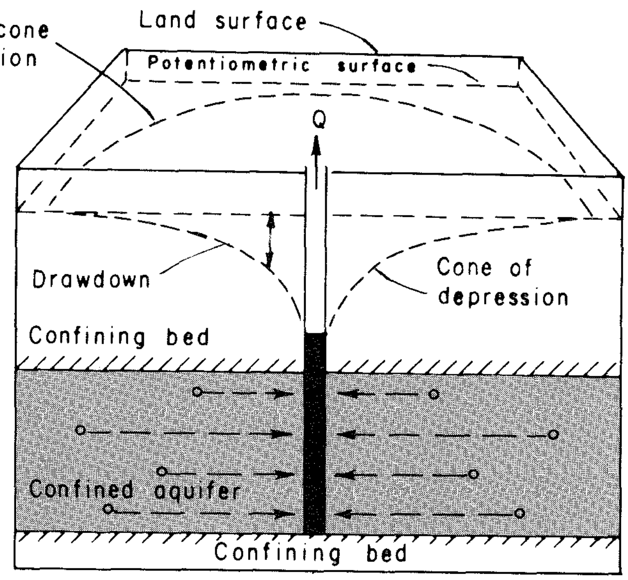


(3)

CONE OF DEPRESSION



(1)



(2)

Both wells and springs serve as sources of ground-water supply. However, most springs having yields large enough to meet municipal, industrial, and large commercial and agricultural needs occur only in areas underlain by cavernous limestones and lava flows. Therefore, most ground-water needs are met by withdrawals from wells.

The response of aquifers to withdrawals from wells is an important topic in ground-water hydrology. When withdrawals start, the water level in the well begins to decline as water is removed from storage in the well. The head in the well falls below the level in the surrounding aquifer. As a result, water begins to move from the aquifer into the well. As pumping continues, the water level in the well continues to decline, and the rate of flow into the well from the aquifer continues to increase until the rate of inflow equals the rate of withdrawal.

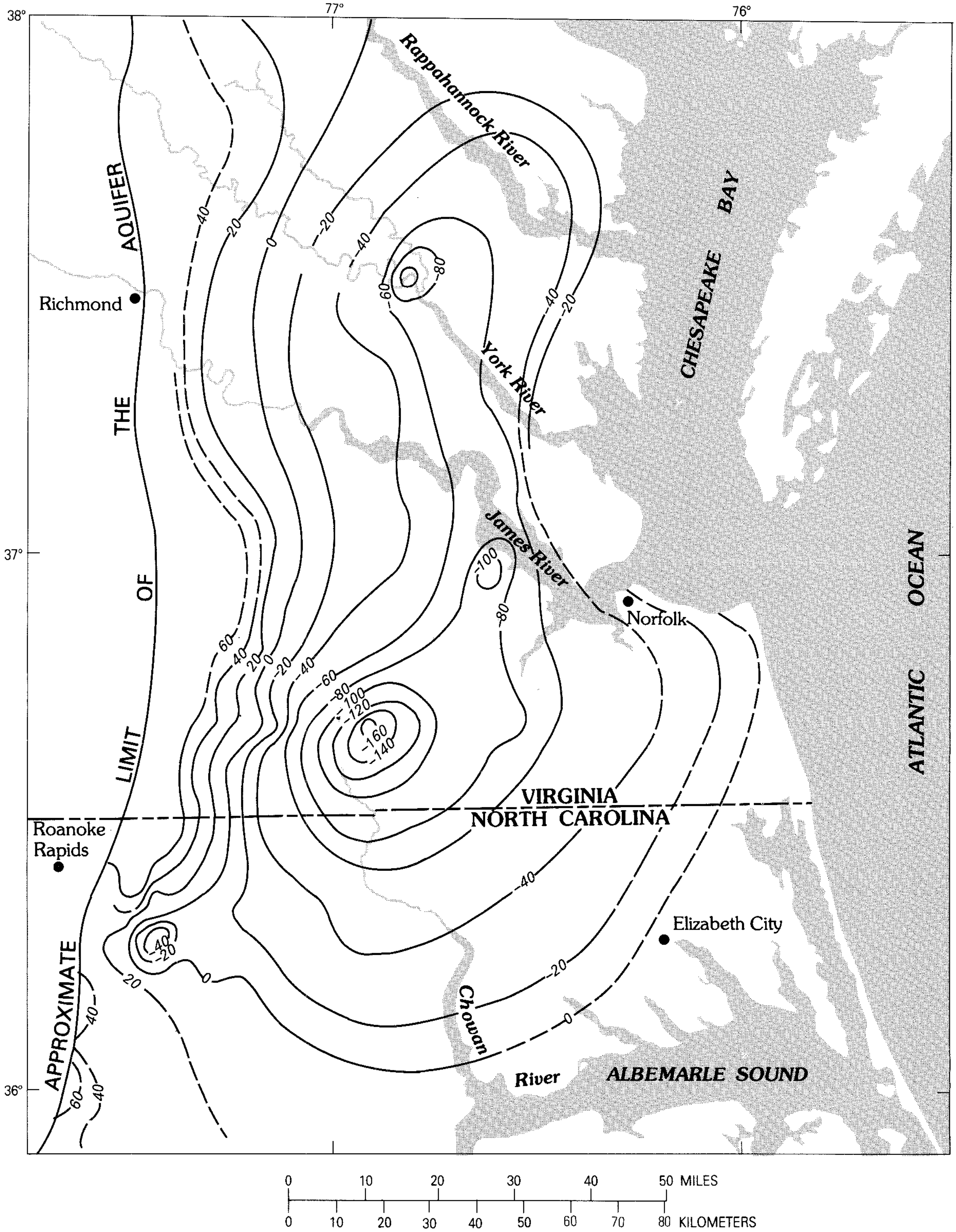
The movement of water from an aquifer into a well results in the formation of a *cone of depression* (1) (2). Because water must converge on the well from all directions and because the area through which the flow occurs decreases toward the well, the hydraulic gradient must get steeper toward the well.

Several important differences exist between the cones of depression in confined and unconfined aquifers. Withdrawals from an unconfined aquifer result in drainage of water from the rocks through which the water table declines as the cone of depression forms (1). Because the storage coefficient of an unconfined aquifer equals the specific yield of the aquifer material, the cone of depression expands very slowly. On the other hand, dewatering of the aquifer results in a decrease in transmissivity, which causes, in turn, an increase in drawdown both in the well and in the aquifer.

Withdrawals from a confined aquifer cause a drawdown in artesian pressure but do not (normally) cause a dewatering of the aquifer (2). The water withdrawn from a confined aquifer is derived from expansion of the water and compression of the rock skeleton of the aquifer. (See "Storage Coefficient.") The very small storage coefficient of confined aquifers results in a very rapid expansion of the cone of depression. Consequently, the mutual interference of expanding cones around adjacent wells occurs more rapidly in confined aquifers than it does in unconfined aquifers.

Cones of depression caused by large withdrawals from extensive confined aquifers can affect very large areas. Sketch 3 shows the overlapping cones of depression that existed in 1981 in an extensive confined aquifer composed of unconsolidated sands and interbedded silt and clay of Cretaceous age in the central part of the Atlantic Coastal Plain. The cones of depression are caused by withdrawals of about 277,000 m^3 d^{-1} (73,000,000 gal d^{-1}) from well fields in Virginia and North Carolina. (See "Source of Water Derived From Wells.")

POTENTIOMETRIC SURFACE OF THE LOWERMOST CRETACEOUS AQUIFER IN SOUTHEASTERN VIRGINIA AND NORTHEASTERN NORTH CAROLINA



EXPLANATION

Water levels are in feet

NATIONAL GEODETIC VERTICAL DATUM 1929

(3)

SOURCE OF WATER DERIVED FROM WELLS

Both the economical development and the effective management of any ground-water system require an understanding of the response of the system to withdrawals from wells. The first concise description of the hydrologic principles involved in this response was presented by C. V. Theis in a paper published in 1940.

Theis pointed out that the response of an aquifer to withdrawals from wells depends on:

1. The rate of expansion of the cone of depression caused by the withdrawals, which depends on the transmissivity and the storage coefficient of the aquifer.
2. The distance to areas in which the rate of water discharging from the aquifer can be reduced.
3. The distance to recharge areas in which the rate of recharge can be increased.

Over a sufficiently long period of time under natural conditions—that is, before the start of withdrawals—the discharge from every ground-water system equals the recharge to it (1). In other words,

$$\text{natural discharge } (D) = \text{natural recharge } (R)$$

In the eastern part of the United States and in the more humid areas in the West, the amount and distribution of precipitation are such that the period of time over which discharge and recharge balance may be less than a year or, at most, a few years. In the drier parts of the country—that is, in the areas that generally receive less than about 500 mm of precipitation annually—the period over which discharge and recharge balance may be several years or even centuries. Over shorter periods of time, differences between discharge and recharge involve changes in ground-water storage. In other words, when discharge exceeds recharge, ground-water storage (S) is reduced by an amount ΔS equal to the difference between discharge and recharge. Thus,

$$D = R + \Delta S$$

Conversely, when recharge exceeds discharge, ground-water storage is increased. Thus,

$$D = R - \Delta S$$

When withdrawal through a well begins, water is removed from storage in its vicinity as the cone of depression develops (2). Thus, the withdrawal (Q) is balanced by a reduction in ground-water storage. In other words,

$$Q = \Delta S$$

As the cone of depression expands outward from the pumping well, it may reach an area where water is discharging from the aquifer. The hydraulic gradient will be reduced toward the discharge area, and the rate of natural discharge will decrease (3). To the extent that the decrease in natural discharge compensates for the pumpage, the rate at which water is being removed from storage will also decrease, and the rate of expansion of the cone of depression will decline. If and when the reduction in natural discharge (ΔD) equals the rate of withdrawal (Q), a new balance will be established in the aquifer. This balance in symbolic form is

$$(D - \Delta D) + Q = R$$

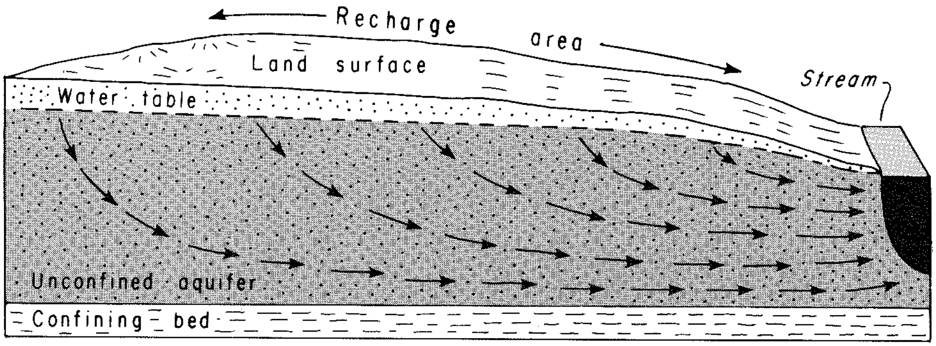
Conversely, if the cone of depression expands into a recharge area rather than into a natural discharge area, the hydraulic gradient between the recharge area and the pumping well will be increased. If, under natural conditions, more water was available in the recharge area than the aquifer could accept (the condition that Theis referred to as one of *rejected recharge*), the increase in the gradient away from the recharge area will permit more recharge to occur, and the rate of growth of the cone of depression will decrease. If and when the increase in recharge (ΔR) equals the rate of withdrawal (Q), a new balance will be established in the aquifer, and expansion of the cone of depression will cease. The new balance in symbolic form is

$$D + Q = R + \Delta R$$

In the eastern part of the United States, gaining streams are relatively closely spaced, and areas in which rejected recharge occurs are relatively unimportant. In this region, the growth of cones of depression first commonly causes a reduction in natural discharge. If the pumping wells are near a stream or if the withdrawals are continued long enough, ground-water discharge to a stream may be stopped entirely in the vicinity of the wells, and water may be induced to move from the stream into the aquifer (4). In other words, the tendency in this region is for withdrawals to change discharge areas into recharge areas. This consideration is important where the streams contain brackish or polluted water or where the streamflow is committed or required for other purposes.

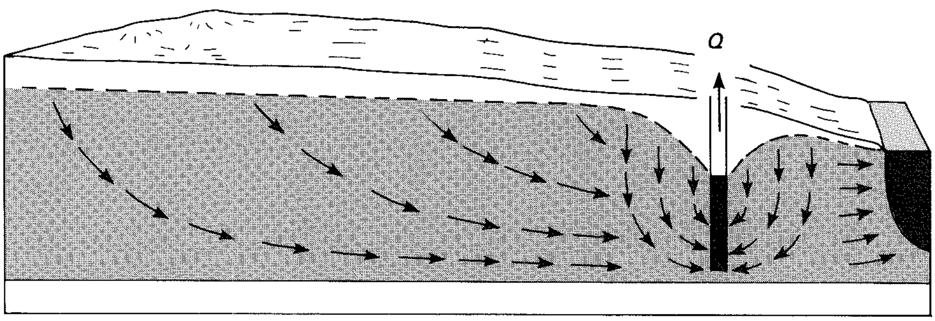
To summarize, the withdrawal of ground water through a well reduces the water in storage in the source aquifer during the growth of the cone of depression. When and if the cone of depression ceases to expand, the rate of withdrawal is being balanced by a reduction in the rate of natural discharge and (or) by an increase in the rate of recharge. Under this condition,

$$Q = \Delta D + \Delta R$$



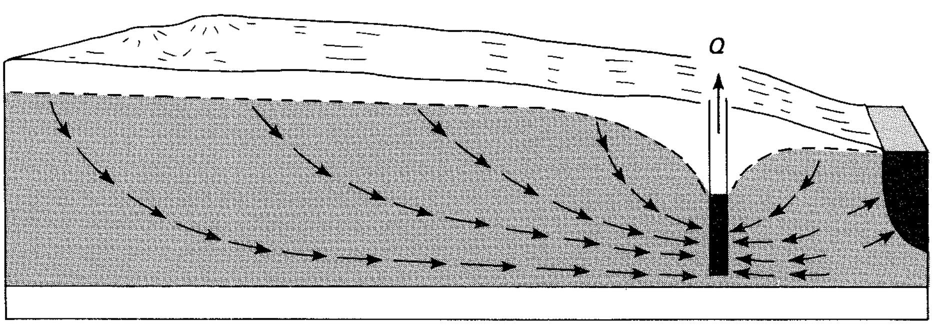
Discharge (D) = Recharge (R)

(1)



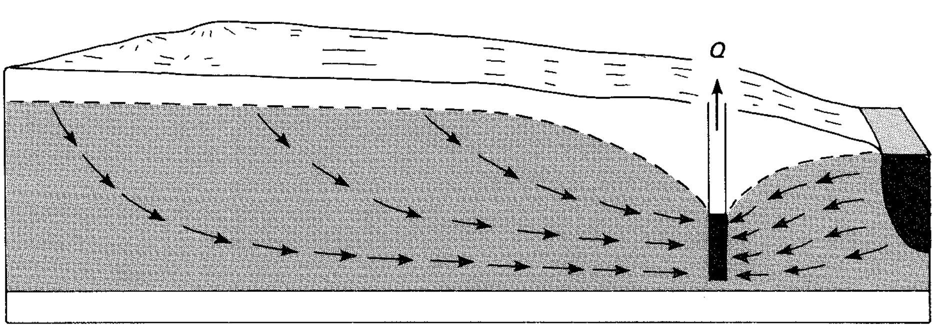
Withdrawal (Q) = Reduction in storage (ΔS)

(2)



Withdrawal (Q) = Reduction in storage (ΔS) + Reduction in discharge (ΔD)

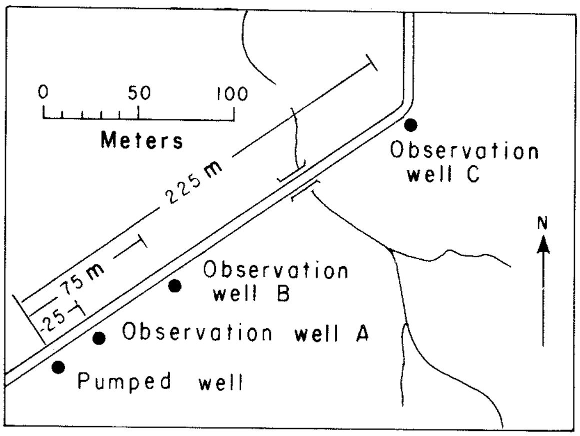
(3)



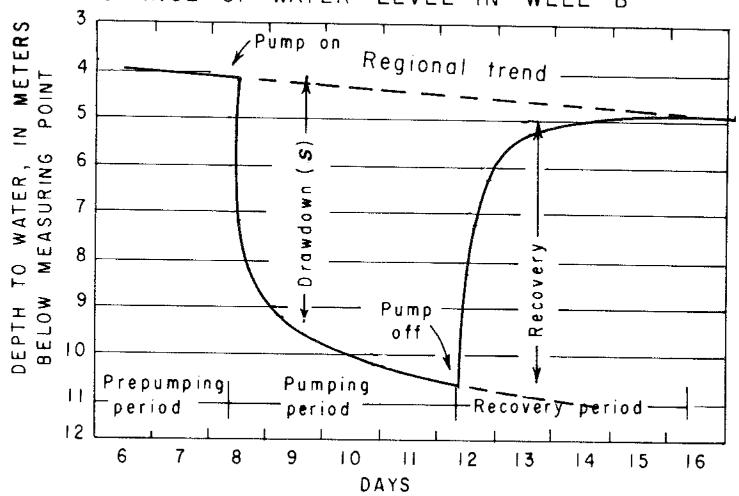
Withdrawal (Q) = Reduction in discharge (ΔD) + Increase in recharge (ΔR)

(4)

AQUIFER TESTS



(1)



(2)

Determining the yield of ground-water systems and evaluating the movement and fate of ground-water pollutants require, among other information, knowledge of:

1. The position and thickness of aquifers and confining beds.
2. The transmissivity and storage coefficient of the aquifers.
3. The hydraulic characteristics of the confining beds.
4. The position and nature of the aquifer boundaries.
5. The location and amounts of ground-water withdrawals.
6. The locations, kinds, and amounts of pollutants and pollutant practices.

Acquiring knowledge on these factors requires both geologic and hydrologic investigations. One of the most important hydrologic studies involves analyzing the change, with time, in water levels (or total heads) in an aquifer caused by withdrawals through wells. This type of study is referred to as an *aquifer test* and, in most cases, includes pumping a well at a constant rate for a period ranging from several hours to several days and measuring the change in water level in observation wells located at different distances from the pumped well (1).

Successful aquifer tests require, among other things:

1. Determination of the prepumping water-level trend (that is, the regional trend).
2. A carefully controlled constant pumping rate.
3. Accurate water-level measurements made at precisely known times during both the drawdown and the recovery periods.

Drawdown is the difference between the water level at any time during the test and the position at which the water level would have been if withdrawals had not started. Drawdown is very rapid at first. As pumping continues and the cone of depression expands, the rate of drawdown decreases (2).

The *recovery* of the water level under ideal conditions is a mirror image of the drawdown. The change in water level during the recovery period is the same as if withdrawals had continued at the same rate from the pumped well but, at the moment of pump cutoff, a recharge well had begun recharging water at the same point and at the same rate. Therefore, the recovery of the water level is the difference between the actual measured level and the projected pumping level (2).

In addition to the constant-rate aquifer test mentioned above, analytical methods have also been developed for several other types of aquifer tests. These methods include tests in which the rate of withdrawal is variable and tests that involve leakage of water across confining beds into confined aquifers. The analytical methods available also permit analysis of tests conducted on both vertical wells and horizontal wells or drains.

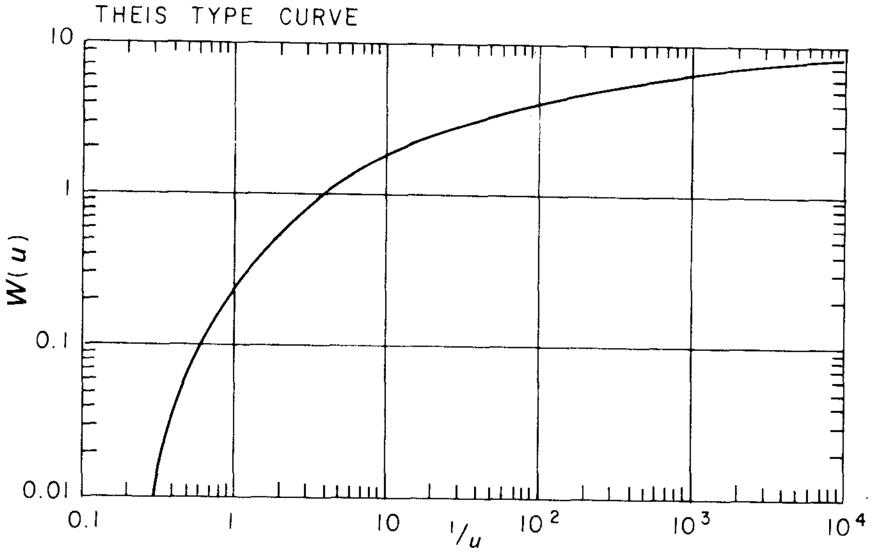
The most commonly used method of analysis of aquifer-test data—that for a vertical well pumped at a constant rate from an aquifer not affected by vertical leakage and lateral boundaries—will be covered in the discussion of "Analysis of Aquifer-Test Data." The method of analysis requires the use of a type curve based on the values of $W(u)$ and $1/u$ listed in the following table. Preparation and use of the type curve are covered in the following discussion.

SELECTED VALUES OF $W(u)$ FOR VALUES OF $1/u$

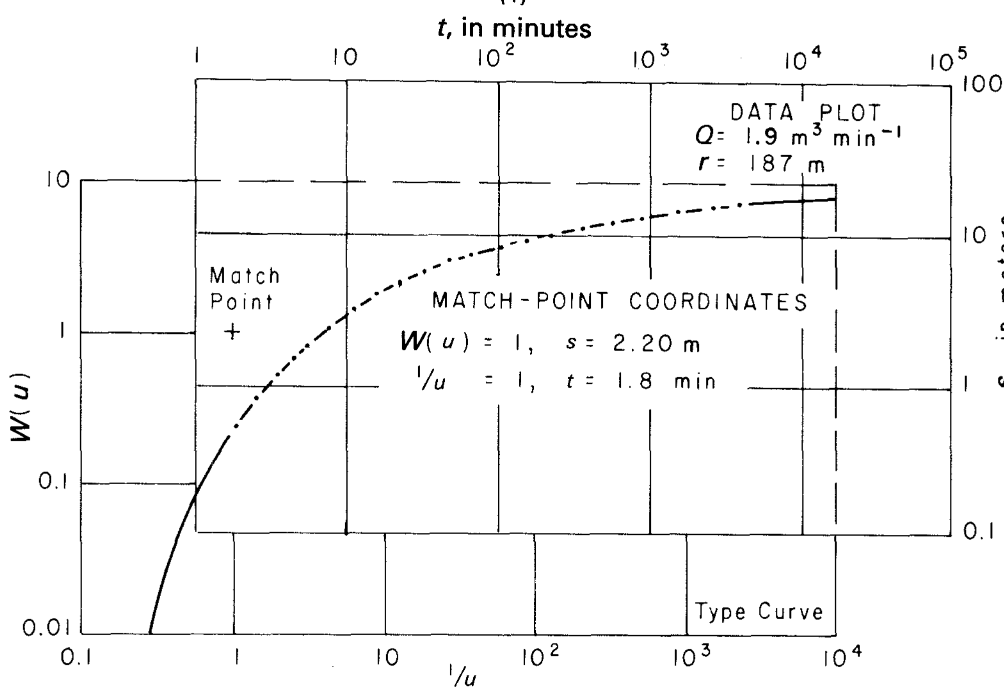
$1/u$	10	7.69	5.88	5.00	4.00	3.33	2.86	2.5	2.22	2.00	1.67	1.43	1.25	1.11
10^{-1}	0.219	0.135	0.075	0.049	0.025	0.013	0.007	0.004	0.002	0.001	0.000	0.000	0.000	0.000
1	1.82	1.59	1.36	1.22	1.04	.91	.79	.70	.63	.56	.45	.37	.31	.26
10	4.04	3.78	3.51	3.35	3.14	2.96	2.81	2.68	2.57	2.47	2.30	2.15	2.03	1.92
10^2	6.33	6.07	5.80	5.64	5.42	5.23	5.08	4.95	4.83	4.73	4.54	4.39	4.26	4.14
10^3	8.63	8.37	8.10	7.94	7.72	7.53	7.38	7.25	7.13	7.02	6.84	6.69	6.55	6.44
10^4	10.94	10.67	10.41	10.24	10.02	9.84	9.68	;9.55	9.43	9.33	9.14	8.99	8.86	8.74
10^5	13.24	12.98	12.71	12.55	12.32	12.14	11.99	11.85	11.73	11.63	11.45	11.29	11.16	11.04
10^6	15.54	15.28	15.01	14.85	14.62	14.44	14.29	14.15	14.04	13.93	13.75	13.60	13.46	13.34
10^7	17.84	17.58	17.31	17.15	16.93	16.74	16.59	16.46	16.34	16.23	16.05	15.90	15.76	15.65
10^8	20.15	19.88	19.62	19.45	19.23	19.05	18.89	18.76	18.64	18.54	18.35	18.20	18.07	17.95
10^9	22.45	22.19	21.92	21.76	21.53	21.35	21.20	21.06	20.94	20.84	20.66	20.50	20.37	20.25
10^{10}	24.75	24.49	24.22	24.06	23.83	23.65	23.50	23.36	23.25	23.14	22.96	22.81	22.67	22.55
10^{11}	27.05	26.79	26.52	26.36	26.14	25.96	25.80	25.67	25.55	25.44	25.26	25.11	24.97	24.86
10^{12}	29.36	20.09	28.83	28.66	28.44	28.26	28.10	27.97	27.85	27.75	27.56	27.41	27.28	27.16
10^{13}	31.66	31.40	31.13	30.97	30.74	30.56	30.41	30.27	30.15	30.05	29.87	29.71	29.58	29.46
10^{14}	33.96	33.70	33.43	33.27	33.05	32.86	32.71	32.58	32.46	32.35	32.17	32.02	31.88	31.76

Examples: When $1/u = 10 \times 10^{-1}$, $W(u) = 0.219$; when $1/u = 3.33 \times 10^2$, $W(u) = 5.23$.

ANALYSIS OF AQUIFER-TEST DATA



(1)



(2)

In 1935, C. V. Theis of the New Mexico Water Resources District of the U.S. Geological Survey developed the first equation to include time of pumping as a factor that could be used to analyze the effect of withdrawals from a well. Thus, the *Theis equation* permitted, for the first time, determination of the hydraulic characteristics of an aquifer before the development of new steady-state conditions resulting from pumping. The importance of this capability may be realized from the fact that, under most conditions, a new steady state cannot be developed or that, if it can, many months or years may be required.

Theis assumed in the development of the equation that:

1. The transmissivity of the aquifer tapped by the pumping well is constant during the test to the limits of the cone of depression.
2. The water withdrawn from the aquifer is derived entirely from storage and is discharged instantaneously with the decline in head.
3. The discharging well penetrates the entire thickness of the aquifer, and its diameter is small in comparison with the pumping rate, so that storage in the well is negligible.

These assumptions are most nearly met by confined aquifers at sites remote from their boundaries. However, if certain precautions are observed, the equation can also be used to analyze tests of unconfined aquifers.

The forms of the Theis equation used to determine the transmissivity and storage coefficient are

$$T=\frac{Q\,W(u)}{4\pi s} \qquad (1)$$

$$S=\frac{4Ttu}{r^2} \qquad (2)$$

where T is transmissivity, S is the storage coefficient, Q is the pumping rate, s is drawdown, t is time, r is the distance from the pumping well to the observation well, $W(u)$ is the well function of u, which equals

$$-0.577216-\log_e u+u-\frac{u^2}{2\times 2!}+\frac{u^3}{3\times 3!}-\frac{u^4}{4\times 4!}+\ldots$$

and $u=(r^2S)/(4Tt)$.

The form of the Theis equation is such that it cannot be solved directly. To overcome this problem, Theis devised a convenient graphic method of solution that involves the use of a type curve (1). To apply this method, a data plot of drawdown versus time (or drawdown versus t/r^2) is matched to the type curve of $W(u)$ versus $1/u$ (2). At some convenient point on the overlapping part of the sheets containing the data plot and type curve, values of s, t (or t/r^2), $W(u)$, and $1/u$ are noted (2). These values are then substituted in equations 1 and 2, which are solved for T and S, respectively.

A Theis type curve of $W(u)$ versus $1/u$ can be prepared from the values given in the table contained in the preceding section, "Aquifer Tests." The data points are plotted on logarithmic graph paper—that is, graph paper having logarithmic divisions in both the x and y directions.

The dimensional units of transmissivity (T) are L^2t^{-1}, where L is length and t is time in days. Thus, if Q in equation 1 is in cubic meters per day and s is in meters, T will be in square meters per day. Similarly, if, in equation 2, T is in square meters per day, t is in days, and r is in meters, S will be dimensionless.

Traditionally, in the United States, T has been expressed in units of gallons per day per foot. The common practice now is to report transmissivity in units of square meters per day or square feet per day. If Q is measured in gallons per minute, as is still normally the case, and drawdown is measured in feet, as is also normally the case, equation 1 is modified to obtain T in square feet per day as follows:

$$T=\frac{Q\,W(u)}{4\pi s}=\frac{\text{gal}}{\text{min}}\times\frac{1{,}440\ \text{min}}{\text{d}}\times\frac{\text{ft}^3}{7.48\ \text{gal}}\times\frac{1}{\text{ft}}\times\frac{W(u)}{4\pi}$$

or

$$T(\text{in ft}^2\ \text{d}^{-1})=\frac{15.3Q\,W(u)}{s}$$

(when Q is in gallons per minute and s is in feet). To convert square feet per day to square meters per day, divide by 10.76.

The storage coefficient is dimensionless. Therefore, if T is in square feet per day, t is in minutes, and r is in feet, then, by equation 2,

$$S=\frac{4Ttu}{r^2}=\frac{4}{1}\times\frac{\text{ft}^2}{\text{d}}\times\frac{\text{min}}{\text{ft}^2}\times\frac{\text{d}}{1{,}440\ \text{min}}$$

or

$$S=\frac{Ttu}{360\,r^2}$$

(when T is in square feet per day, t is in minutes, and r is in feet).

Analysis of aquifer-test data using the Theis equation involves plotting both the type curve and the test data on logarithmic graph paper. If the aquifer and the conditions of the test satisfy Theis's assumptions, the type curve has the same shape as the cone of depression along any line radiating away from the pumping well and the drawdown graph at any point in the cone of depression.

Use of the Theis equation for unconfined aquifers involves two considerations. First, if the aquifer is relatively fine grained, water is released slowly over a period of hours or days, not instantaneously with the decline in head. Therefore, the value of S determined from a short-period test may be too small.

Second, if the pumping rate is large and the observation well is near the pumping well, dewatering of the aquifer may be significant, and the assumption that the transmissivity of the aquifer is constant is not satisfied. The effect of dewatering of the aquifer can be eliminated with the following equation:

$$s'=s-\left(\frac{s^2}{2b}\right) \qquad (3)$$

where s is the observed drawdown in the unconfined aquifer, b is the aquifer thickness, and s' is the drawdown that would have occurred if the aquifer had been confined (that is, if no dewatering had occurred).

To determine the transmissivity and storage coefficient of an unconfined aquifer, a data plot consisting of s' versus t (or t/r^2) is matched with the Theis type curve of $W(u)$ versus $1/u$. Both s and b in equation 3 must be in the same units, either feet or meters.

As noted above, Theis assumed in the development of his equation that the discharging well penetrates the entire thickness of the aquifer. However, because it is not always possible, or necessarily desirable, to design a well that fully penetrates the aquifer under development, most discharging wells are open to only a part of the aquifer that they draw from. Such *partial penetration* creates vertical flow in the vicinity of the discharging well that may affect drawdowns in observation wells located relatively close to the discharging well. Drawdowns in observation wells that are open to the same zone as the discharging well will be larger than the drawdowns in wells at the same distance from the discharging well but open to other zones. The possible effect of partial penetration on drawdowns must be considered in the analysis of aquifer-test data. If aquifer-boundary and other conditions permit, the problem can be avoided by locating observation wells beyond the zone in which vertical flow exists.

TIME-DRAWDOWN ANALYSIS

The Theis equation is only one of several methods that have been developed for the analysis of aquifer-test data. (See "Analysis of Aquifer-Test Data.") Another method, and one that is somewhat more convenient to use, was developed by C. E. Jacob from the Theis equation. The greater convenience of the Jacob method derives partly from its use of semilogarithmic graph paper instead of the logarithmic paper used in the Theis method and from the fact that, under ideal conditions, the data plot along a straight line rather than along a curve.

However, it is essential to note that, whereas the Theis equation applies at all times and places (if the assumptions are met), Jacob's method applies only under certain additional conditions. These conditions must also be satisfied in order to obtain reliable answers.

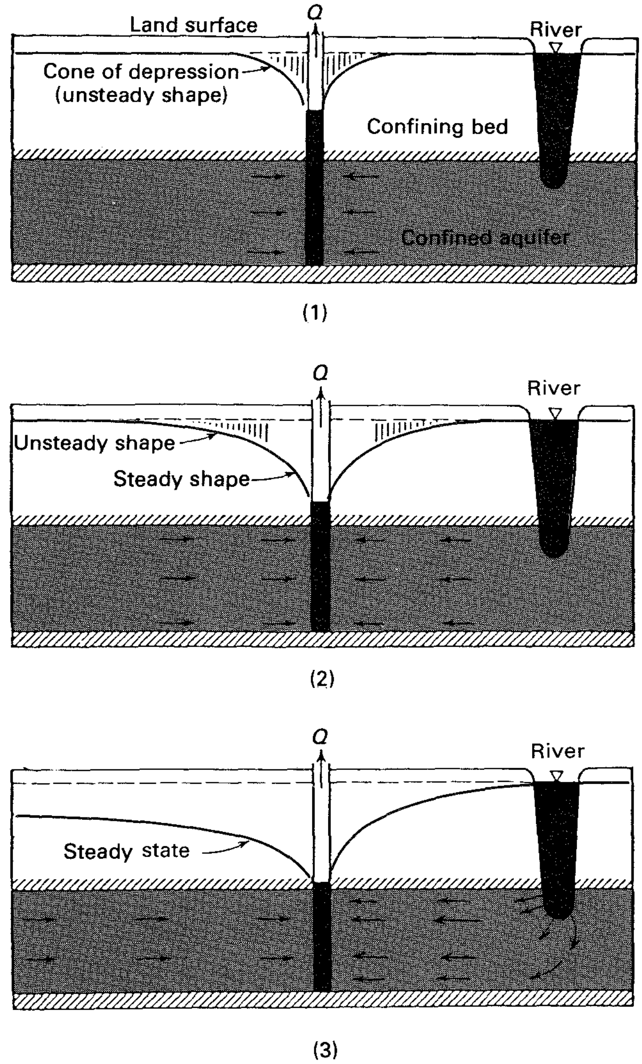
To understand the limitations of Jacob's method, we must consider the changes that occur in the cone of depression during an aquifer test. The changes that are of concern involve both the *shape of the cone* and the *rate of drawdown*. As the cone of depression migrates outward from a pumping well, its shape (and, therefore, the hydraulic gradient at different points in the cone) changes. We can refer to this condition as *unsteady shape*. At the start of withdrawals, the entire cone of depression has an unsteady shape (1). After a test has been underway for some time, the cone of depression begins to assume a relatively *steady shape*, first at the pumping well and then gradually to greater and greater distances (2). If withdrawals continue long enough for increases in recharge and (or) reductions in discharge to balance the rate of withdrawal, drawdowns cease, and the cone of depression is said to be in a *steady state* (3).

The Jacob method is applicable only to the zone in which steady-shape conditions prevail or to the entire cone only after steady-state conditions have developed. For practical purposes, this condition is met when $u=(r^2S)/(4Tt)$ is equal to or less than about 0.05. Substituting this value in the equation for u and solving for t, we can determine the time at which steady-shape conditions develop at the outermost observation well. Thus,

$$t_c = \frac{7{,}200\, r^2S}{T} \quad (1)$$

where t_c is the time, in minutes, at which steady-shape conditions develop, r is the distance from the pumping well, in feet (or meters), S is the estimated storage coefficient (dimensionless), and T is the estimated transmissivity, in square feet per day (or square meters per day).

After steady-shape conditions have developed, the drawdowns at an observation well begin to fall along a straight line on semilogarithmic graph paper, as sketch 4 shows. Before that time, the drawdowns plot below the extension of the straight line. When a time-drawdown graph is prepared, drawdowns are plotted on the vertical (arithmetic) axis versus time on the horizontal (logarithmic) axis.

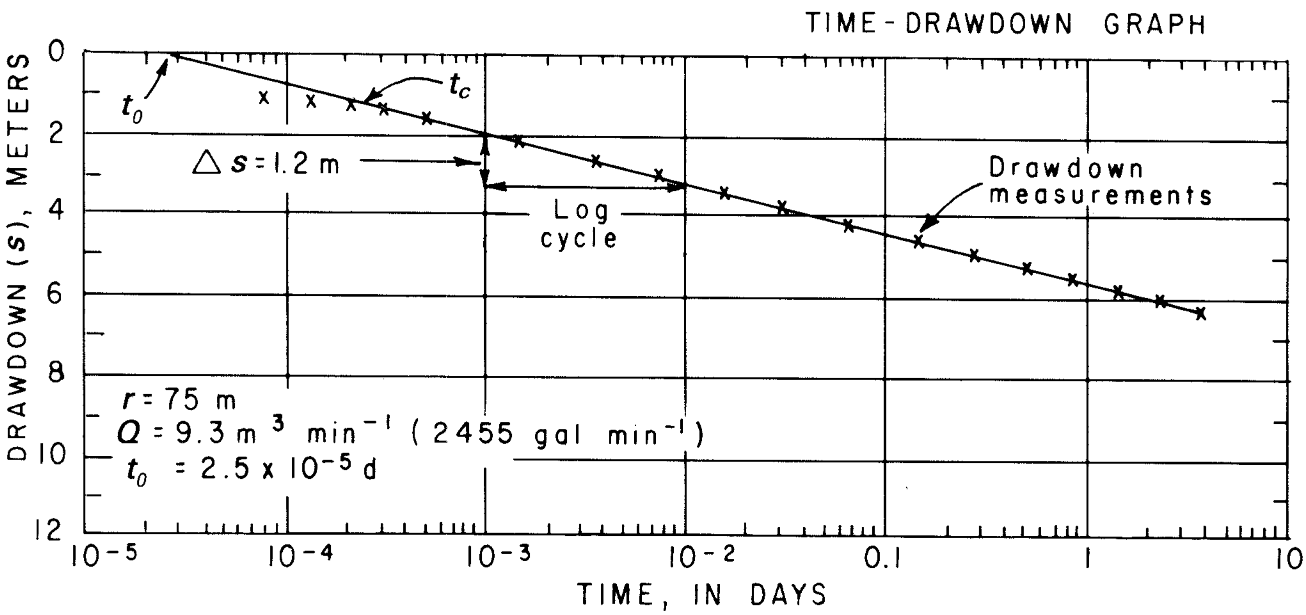


The slope of the straight line is proportional to the pumping rate and to the transmissivity. Jacob derived the following equations for determination of transmissivity and storage coefficient from the time-drawdown graphs:

$$T = \frac{2.3\, Q}{4\pi \Delta s} \quad (2)$$

$$S = \frac{2.25\, Tt_0}{r^2} \quad (3)$$

where Q is the pumping rate, Δs is the drawdown across one log cycle, t_0 is the time at the point where the straight line intersects the zero-drawdown line, and r is the distance from the pumping well to the observation well.



(4)

Equations 2 and 3 are in consistent units. Thus, if Q is in cubic meters per day and s is in meters, T is in square meters per day. S is dimensionless, so that, in equation 3, if T is in square meters per day, then r must be in meters and t_0 must be in days.

It is still common practice in the United States to express Q in gallons per minute, s in feet, t in minutes, r in feet, and T in square feet per day. We can modify equations 2 and 3 for direct substitution of these units as follows:

$$T = \frac{2.3\,Q}{4\pi\Delta s} = \frac{2.3}{4\pi} \times \frac{\text{gal}}{\text{min}} \times \frac{1{,}440\ \text{min}}{\text{d}} \times \frac{\text{ft}^3}{7.48\ \text{gal}} \times \frac{1}{\text{ft}}$$

$$T = \frac{35\,Q}{\Delta s} \tag{4}$$

(where T is in square feet per day, Q is in gallons per minute, and Δs is in feet) and

$$S = \frac{2.25\,Tt_0}{r^2} = \frac{2.25}{1} \times \frac{\text{ft}^2}{\text{d}} \times \frac{\text{min}}{\text{ft}^2} \times \frac{\text{d}}{1{,}440\ \text{min}}$$

$$S = \frac{Tt_0}{640\,r^2} \tag{5}$$

(where T is in square feet per day, t_0 is in minutes, and r is in feet).

DISTANCE-DRAWDOWN ANALYSIS

It is desirable in aquifer tests to have at least three observation wells located at different distances from the pumping well (1). Drawdowns measured at the same time in these wells can be analyzed with the Theis equation and type curve to determine the aquifer transmissivity and storage coefficient.

After the test has been underway long enough, drawdowns in the wells can also be analyzed by the Jacob method, either through the use of a time-drawdown graph using data from individual wells or through the use of a distance-drawdown graph using "simultaneous" measurements in all of the wells. To determine when sufficient time has elapsed, see "Time-Drawdown Analysis."

In the Jacob distance-drawdown method, drawdowns are plotted on the vertical (arithmetic) axis versus distance on the horizontal (logarithmic) axis (2). If the aquifer and test conditions satisfy the Theis assumptions and the limitation of the Jacob method, the drawdowns measured at the same time in different wells should plot along a straight line (2).

The slope of the straight line is proportional to the pumping rate and to the transmissivity. Jacob derived the following equations for determination of the transmissivity and storage coefficient from distance-drawdown graphs:

$$T = \frac{2.3Q}{2\pi\Delta s} \tag{1}$$

$$S = \frac{2.25Tt}{r_0^2} \tag{2}$$

where Q is the pumping rate, Δs is the drawdown across one log cycle, t is the time at which the drawdowns were measured, and r_0 is the distance from the pumping well to the point where the straight line intersects the zero-drawdown line.

Equations 1 and 2 are in consistent units. For the inconsistent units still in relatively common use in the United States, equations 1 and 2 should be used in the following forms:

$$T = \frac{70\,Q}{\Delta s} \tag{3}$$

(where T is in square feet per day, Q is in gallons per minute, and Δs is in feet) and

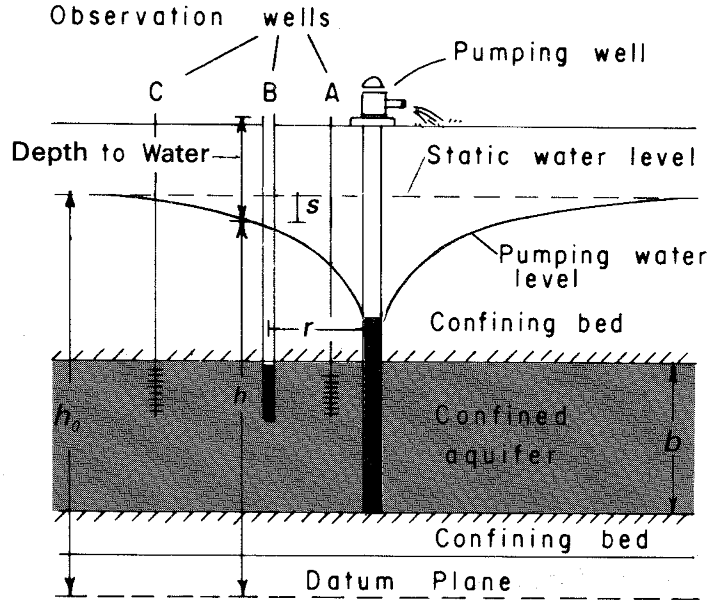
$$S = \frac{Tt}{640\,r_0^2} \tag{4}$$

(where T is in square feet per day, t is in minutes, and r_0 is in feet).

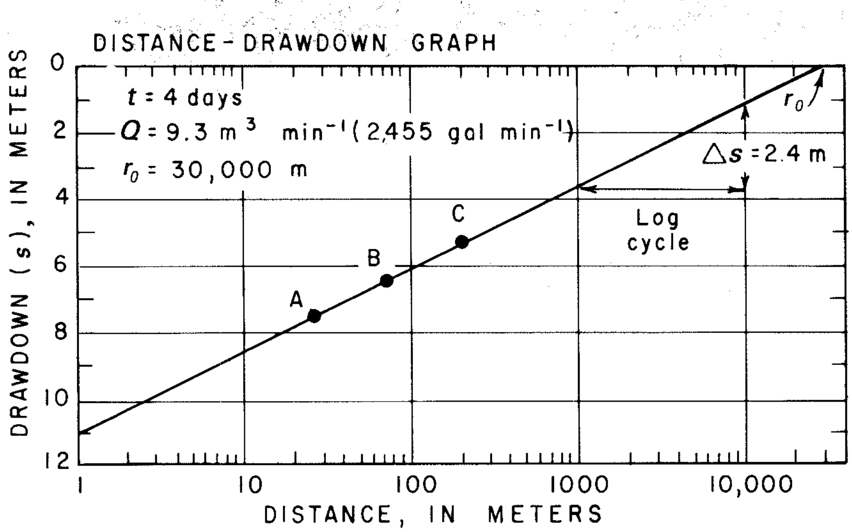
The distance r_0 does not indicate the outer limit of the cone of depression. Because nonsteady-shape conditions exist in the outer part of the cone, before the development of steady-state conditions, the Jacob method does not apply to that part. If the Theis equation were used to calculate drawdowns in the outer part of the cone, it would be found that they would plot below the straight line. In other words, the measurable limit of the cone of depression is beyond the distance r_0.

If the straight line of the distance-drawdown graph is extended inward to the radius of the pumping well, the drawdown indicated at that point is the drawdown in the aquifer outside of the well. If the drawdown inside the well is found to be greater than the drawdown outside, the difference is attributable to *well loss*. (See "Single-Well Tests.")

As noted in the section on "Hydraulic Conductivity," the hydraulic conductivities and, therefore, the transmissivities of aquifers may be different in different directions. These differences may cause drawdowns measured at the same time in observation wells located at the same distances but in different directions from the discharging well to be different. Where this condition exists, the distance-drawdown method may yield satisfactory results only where three or more observation wells are located in the same direction but at different distances from the discharging well.

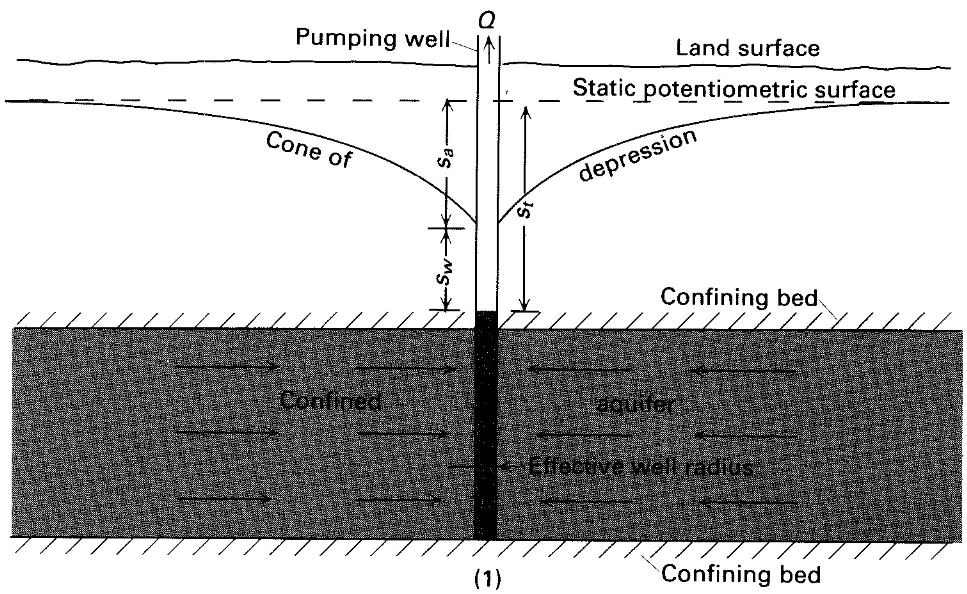

Observation wells
C
B
A
Pumping well
Depth to Water
Static water level
s
Pumping water level
r
Confining bed
h
h_0
Confined aquifer
b
Confining bed
Datum Plane

(1)


DISTANCE - DRAWDOWN GRAPH
t = 4 days
Q = 9.3 m³ min⁻¹ (2,455 gal min⁻¹)
r_0 = 30,000 m
r_0
Δs = 2.4 m
Log cycle
A
B
C
DRAWDOWN (s), IN METERS
0
2
4
6
8
10
12
1
10
100
1000
10,000
DISTANCE, IN METERS

(2)

SINGLE-WELL TESTS



(1)

The most useful aquifer tests are those that include water-level measurements in observation wells. Such tests are commonly referred to as *multiple-well tests*. It is also possible to obtain useful data from production wells, even where observation wells are not available. Such tests are referred to as *single-well tests* and may consist of pumping a well at a single constant rate, or at two or more different but constant rates (see "Well-Acceptance Tests and Well Efficiency") or, if the well is not equipped with a pump, by "instantaneously" introducing a known volume of water into the well. This discussion will be limited to tests involving a single constant rate.

In order to analyze the data, it is necessary to understand the nature of the drawdown in a pumping well. The total drawdown (s_t) in most, if not all, pumping wells consists of two components (1). One is the drawdown (s_a) in the aquifer, and the other is the drawdown (s_w) that occurs as water moves from the aquifer into the well and up the well bore to the pump intake. Thus, the drawdown in most pumping wells is greater than the drawdown in the aquifer at the radius of the pumping well.

The total drawdown (s_t) in a pumping well can be expressed in the form of the following equations:

$$s_t = s_a + s_w$$

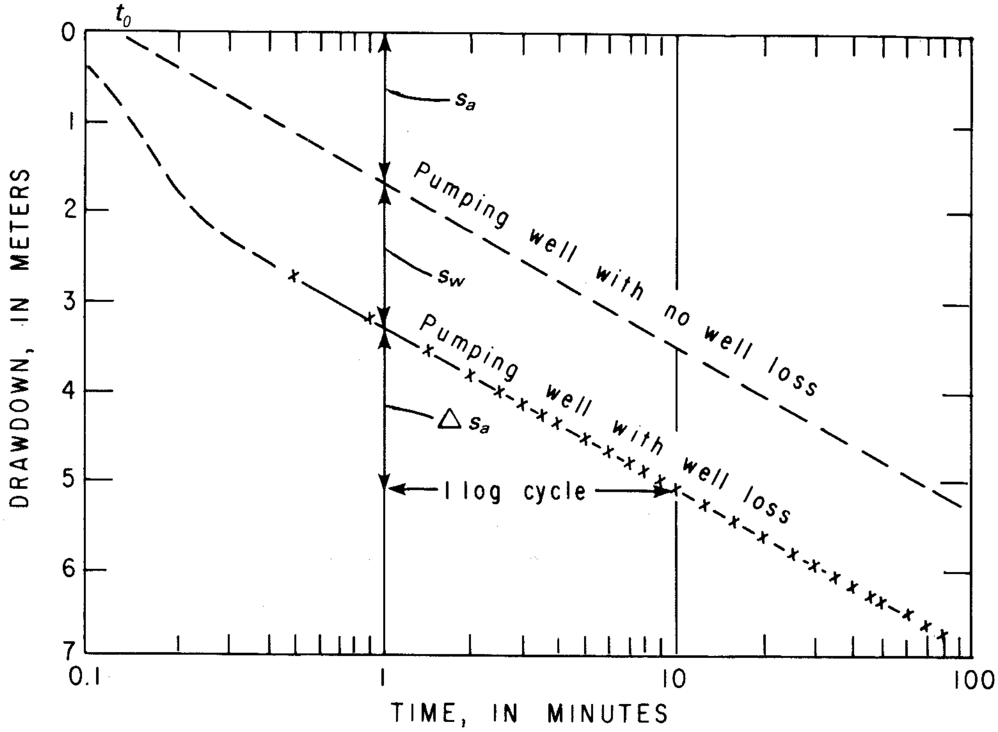
$$s_t = BQ + CQ^2 \tag{1}$$

where s_a is the drawdown in the aquifer at the effective radius of the pumping well, s_w is well loss, Q is the pumping rate, B is a factor related to the hydraulic characteristics of the aquifer and the length of the pumping period, and C is a factor related to the characteristics of the well.

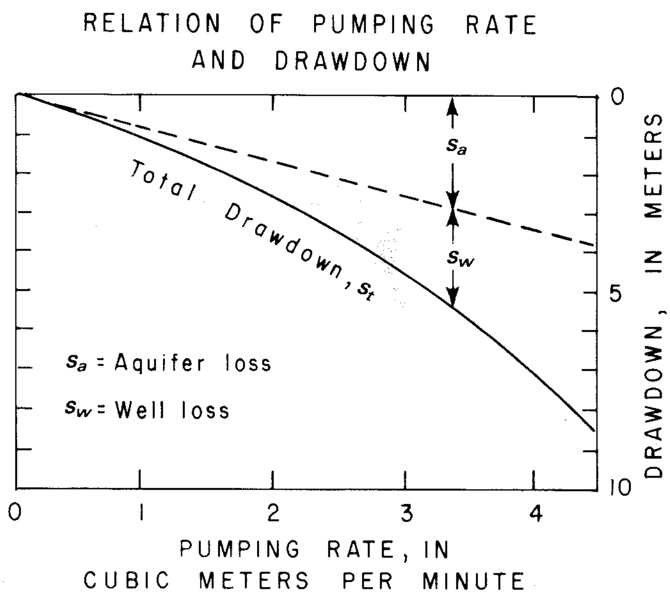
The factor C in equation 1 is normally considered to be constant, so that, in a constant rate test, CQ^2 is also constant. As a result, the well loss (s_w) increases the total drawdown in the pumping well but does not affect the rate of change in the drawdown with time. It is, therefore, possible to analyze drawdowns in the pumping well with the Jacob time-drawdown method using semilogarithmic graph paper. (See "Time-Drawdown Analysis.") Drawdowns are plotted on the arithmetic scale versus time on the logarithmic scale (2), and transmissivity is determined from the slope of the straight line through the use of the following equation:

$$T = \frac{2.3Q}{4\pi\Delta s} \tag{2}$$

Where well loss is present in the pumping well, the storage coefficient cannot be determined by extending the straight line to the line of zero drawdown. Even where well loss is not present, the determination of the storage coefficient from drawdowns in a pumping well likely will be subject to large error because the effective radius of the well may differ significantly from the "nominal" radius.



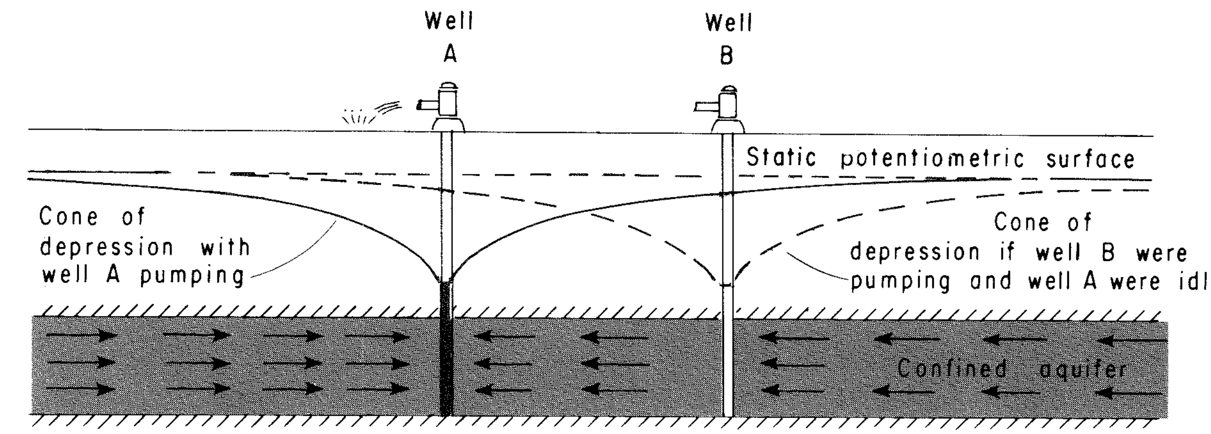
(2)



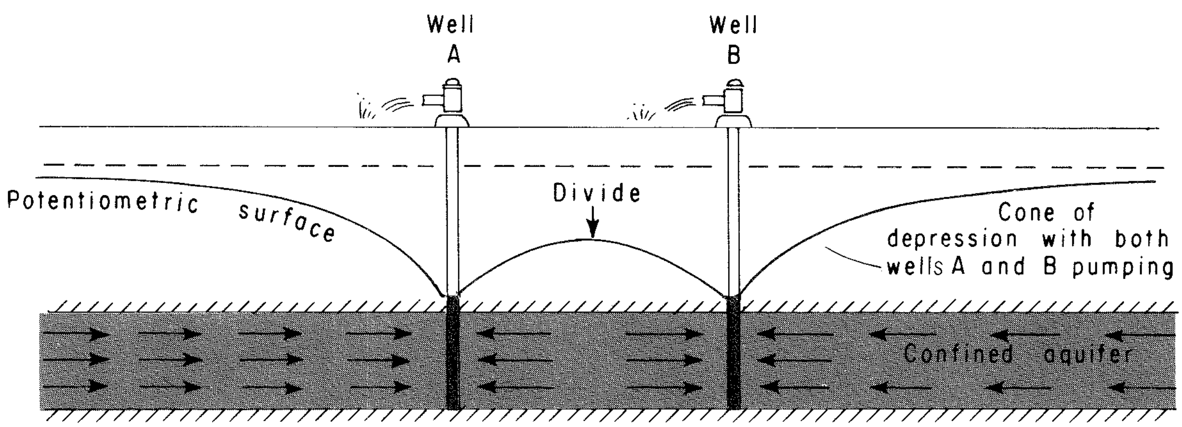
(3)

In equation 1, drawdown in the pumping well is proportional to the pumping rate. The factor B in the aquifer-loss term (BQ) increases with time of pumping as long as water is being derived from storage in the aquifer. The factor C in the well-loss term (CQ^2) is a constant if the characteristics of the well remain unchanged, but, because the pumping rate in the well-loss term is squared, drawdown due to well loss increases rapidly as the pumping rate is increased. The relation between pumping rates and drawdown in a pumping well, if the well was pumped for the same length of time at each rate, is shown in sketch 3. The effect of well loss on drawdown in the pumping well is important both in the analysis of data from pumping wells and in the design of supply wells.

WELL INTERFERENCE



(1)



(2)

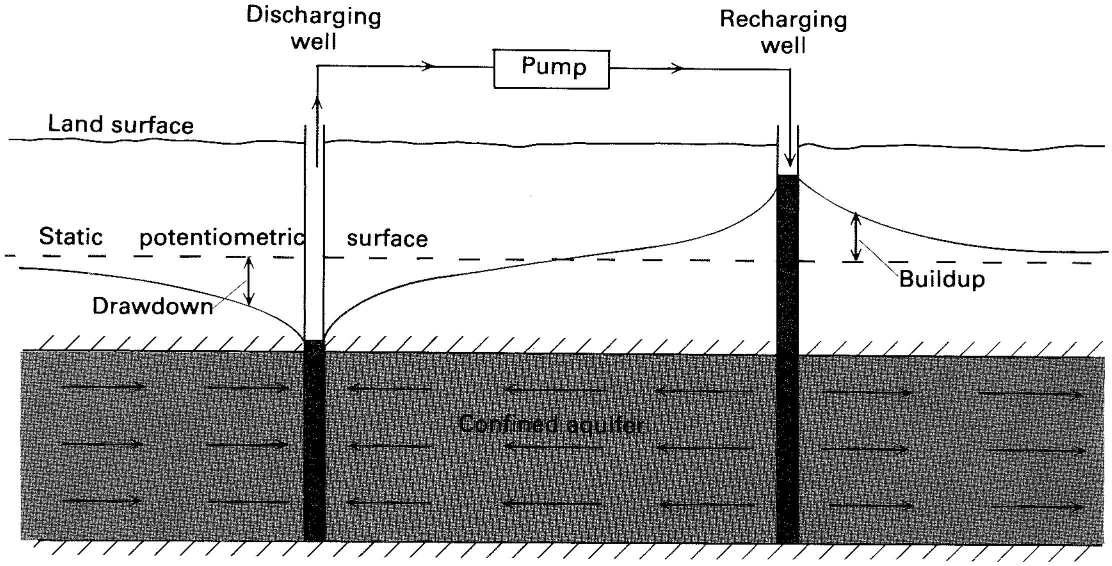
Pumping a well causes a drawdown in the ground-water level in the surrounding area. The drawdown in water level forms a conical-shaped depression in the water table or potentiometric surface, which is referred to as a *cone of depression*. (See "Cone of Depression.") Similarly, a well through which water is injected into an aquifer (that is, a recharge or injection well) causes a buildup in ground-water level in the form of a conical-shaped mound.

The drawdown (s) in an aquifer caused by pumping at any point in the aquifer is directly proportional to the pumping rate (Q) and the length of time (t) that pumping has been in progress and is inversely proportional to the transmissivity (T), the storage coefficient (S), and the square of the distance (r^2) between the pumping well and the point. In other words,

$$s \approx \frac{Q,t}{T,S,r^2} \qquad (1)$$

Where pumping wells are spaced relatively close together, pumping of one will cause a drawdown in the others. Drawdowns are additive, so that the total drawdown in a pumping well is equal to its own drawdown plus the drawdowns caused at its location by other pumping wells (1) (2). The drawdowns in pumping wells caused by withdrawals from other pumping wells are referred to as *well interference*. As sketch 2 shows, a divide forms in the potentiometric surface (or the water table, in the case of an unconfined aquifer) between pumping wells.

At any point in an aquifer affected by both a discharging well and a recharging well, the change in water level is equal to the difference between the drawdown and the buildup. If the rates of discharge and recharge are the same and if the wells are operated on the same schedule, the drawdown and the buildup will cancel midway between the wells, and the water level at that point will remain unchanged from the static level (3). (See "Aquifer Boundaries.")



(3)

We see from the above functional equation that, in the absence of well interference, drawdown in an aquifer at the effective radius of a pumping well is directly proportional to the pumping rate. Conversely, the maximum pumping rate is directly proportional to the *available drawdown*. For confined aquifers, available drawdown is normally considered to be the distance between the prepumping water level and the top of the aquifer. For unconfined aquifers, available drawdown is normally considered to be about 60 percent of the saturated aquifer thickness.

Where the pumping rate of a well is such that only a part of the available drawdown is utilized, the only effect of well interference is to lower the pumping level and, thereby, increase pumping costs. In the design of a well field, the increase in pumping cost must be evaluated along with the cost of the additional waterlines and powerlines that must be installed if the spacing of wells is increased to reduce well interference. (See "Well-Field Design.")

Because well interference reduces the available drawdown, it also reduces the maximum yield of a well. Well interference is, therefore, an important matter in the design of well fields where it is desirable for each well to be pumped at the largest possible rate. We can see from equation 1 that, for a group of wells pumped at the same rate and on the same schedule, the well interference caused by any well on another well in the group is inversely proportional to the square of the distance between the two wells (r^2). Therefore, excessive well interference is avoided by increasing the spacing between wells and by locating the wells along a line rather than in a circle or in a grid pattern.

AQUIFER BOUNDARIES

One of the assumptions inherent in the Theis equation (and in most other fundamental ground-water flow equations) is that the aquifer to which it is being applied is infinite in extent. Obviously, no such aquifer exists on Earth. However, many aquifers are areally extensive, and, because pumping will not affect recharge or discharge significantly for many years, most water pumped is from ground-water storage; as a consequence, water levels must decline for many years. An excellent example of such an aquifer is that underlying the High Plains from Texas to South Dakota.

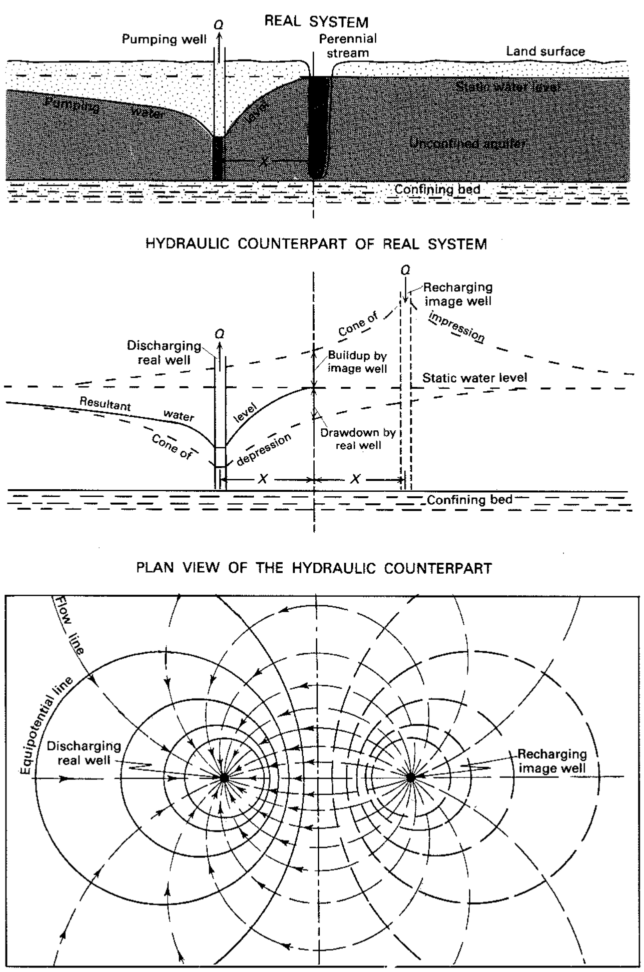
All aquifers are bounded in both the vertical direction and the horizontal direction. For example, vertical boundaries may include the water table, the plane of contact between each aquifer and each confining bed, and the plane marking the lower limit of the zone of interconnected openings—in other words, the base of the ground-water system.

Hydraulically, aquifer boundaries are of two types: recharge boundaries and impermeable boundaries. A *recharge boundary* is a boundary along which flow lines originate. In other words, such a boundary will, under certain hydraulic conditions, serve as a source of recharge to the aquifer. Examples of recharge boundaries include the zones of contact between an aquifer and a perennial stream that completely penetrates the aquifer or the ocean.

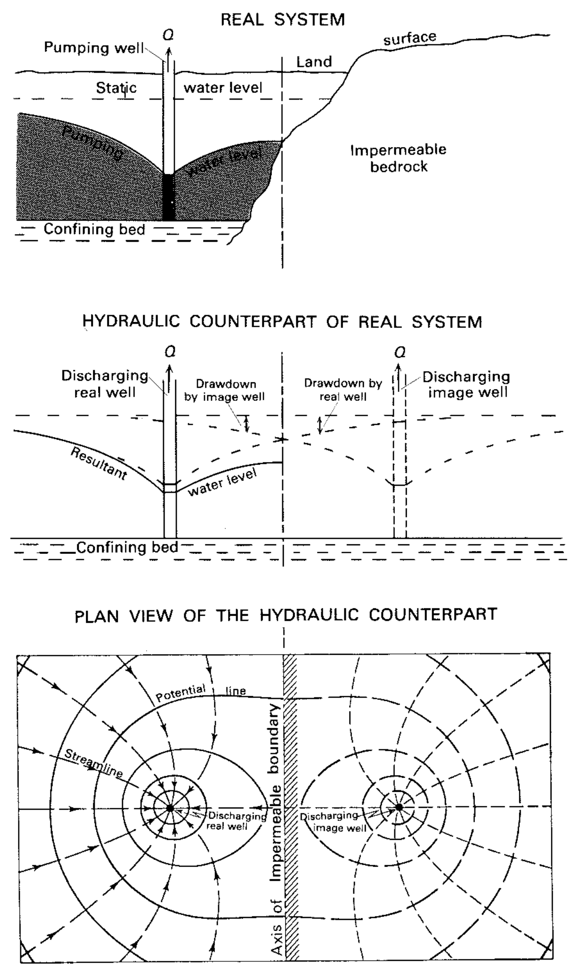
An *impermeable boundary* is a boundary that flow lines do not cross. Such boundaries exist where aquifers terminate against "impermeable" material. Examples include the contact between an aquifer composed of sand and a laterally adjacent bed composed of clay.

The position and nature of aquifer boundaries are of critical importance in many ground-water problems, including the movement and fate of pollutants and the response of aquifers to withdrawals. Depending on the direction of the hydraulic gradient, a stream, for example, may be either the source or the destination of a pollutant.

Lateral boundaries within the cone of depression have a profound effect on the response of an aquifer to withdrawals. To analyze, or to predict, the effect of a lateral boundary, it is necessary to "make" the aquifer appear to be of infinite extent. This feat is accomplished through the use of imaginary



(1)



(2)

wells and the *theory of images*. Sketches 1 and 2 show, in both plan view and profile, how image wells are used to compensate, hydraulically, for the effects of both recharging and impermeable boundaries. (See "Well Interference.")

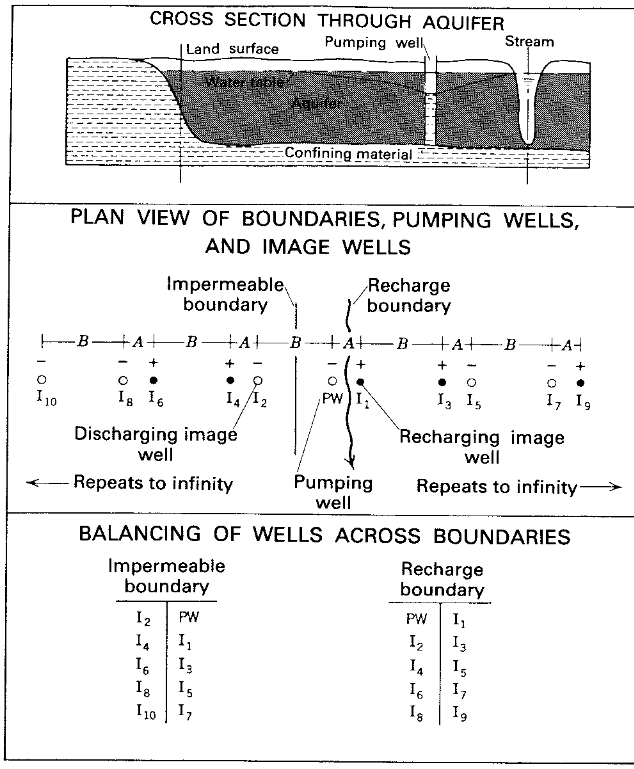
The key feature of a recharge boundary is that withdrawals from the aquifer do not produce drawdowns across the boundary. A perennial stream in intimate contact with an aquifer represents a recharge boundary because pumping from the aquifer will induce recharge from the stream. The hydraulic effect of a recharge boundary can be duplicated by assuming that a recharging image well is present on the side of the boundary opposite the real discharging well. Water is injected into the image well at the same rate and on the same schedule that water is withdrawn from the real well. In the plan view in sketch 1, flow lines originate at the boundary, and equipotential lines parallel the boundary at the closest point to the pumping (real) well.

The key feature of an impermeable boundary is that no water can cross it. Such a boundary, sometimes termed a "no-flow boundary," resembles a divide in the water table or the potentiometric surface of a confined aquifer. The effect of an impermeable boundary can be duplicated by assuming that a discharging image well is present on the side of the boundary opposite the real discharging well. The image well withdraws water at the same rate and on the same schedule as the real well. Flow lines tend to be parallel to an impermeable boundary, and equipotential lines intersect it at a right angle.

The image-well theory is an essential tool in the design of well fields near aquifer boundaries. Thus, on the basis of minimizing the lowering of water levels, the following conditions apply:

1. Pumping wells should be located parallel to and as close as possible to recharging boundaries.
2. Pumping wells should be located perpendicular to and as far as possible from impermeable boundaries.

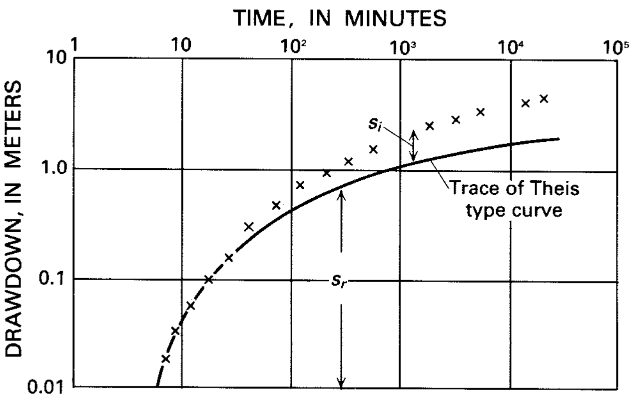
Sketches 1 and 2 illustrate the effect of single boundaries and show how their hydraulic effect is compensated for through the use of single image wells. It is assumed in these sketches that other boundaries are so remote that they have a negligible effect on the areas depicted. At many places, however, pumping wells are affected by two or more boundaries. One example is an alluvial aquifer composed of sand and gravel bordered on one side by a perennial stream (a recharge boundary) and on the other by impermeable bedrock (an impermeable boundary).



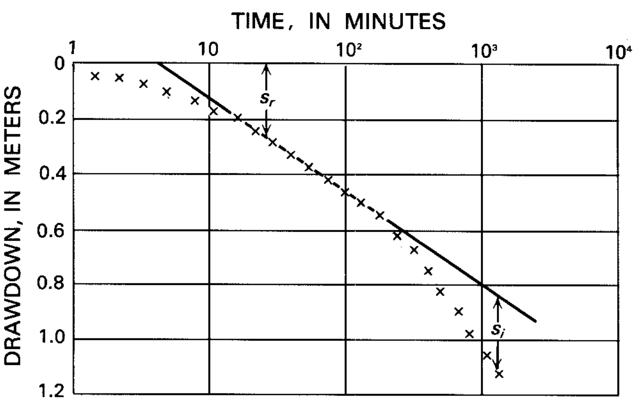
(3)

Contrary to first impression, these boundary conditions cannot be satisfied with only a recharging image well and a discharging image well. Additional image wells are required, as sketch 3 shows, to compensate for the effect of the image wells on the opposite boundaries. Because each new image well added to the array affects the opposite boundary, it is necessary to continue adding image wells until their distances from the boundaries are so great that their effect becomes negligible.

TESTS AFFECTED BY LATERAL BOUNDARIES



(1)

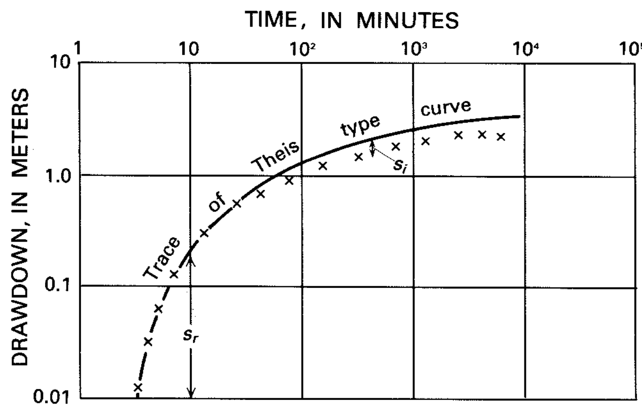


(2)

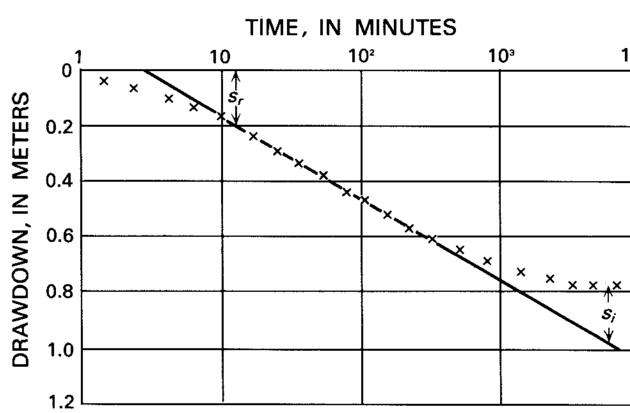
When an aquifer test is conducted near one of the lateral boundaries of an aquifer, the drawdown data depart from the Theis type curve and from the initial straight line produced by the Jacob method. The hydraulic effect of lateral boundaries is assumed, for analytical convenience, to be due to the presence of other wells. (See "Aquifer Boundaries.") Thus, a recharge boundary has the same effect on drawdowns as a recharging image well located across the boundary and at the same distance from the boundary as the real well. The image well is assumed to operate on the same schedule and at the same rate as the real well. Similarly, an impermeable boundary has the same effect on drawdowns as a discharging image well.

To analyze aquifer-test data affected by either a recharge boundary or an impermeable boundary, the early drawdown data in the observation wells nearest the pumping well must not be affected by the boundary. These data, then, show only the effect of the real well and can be used to determine the transmissivity (*T*) and the storage coefficient (*S*) of the aquifer. (See "Analysis of Aquifer-Test Data" and "Time-Drawdown Analysis.") In the Theis method, the type curve is matched to the early data, and a "match point" is selected for use in calculating values of *T* and *S*. The position of the type curve, in the region where the drawdowns depart from the type curve, is traced onto the data plot (1) (3). The trace of the type curve shows where the drawdowns would have plotted if there had been no boundary effect. The differences in drawdown between the data plot and the trace of the type curve show the effect of an aquifer boundary. The direction in which the drawdowns depart from the type curve—that is, in the direction of either greater drawdowns or lesser drawdowns—shows the type of boundary.

Drawdowns greater than those defined by the trace of the type curve indicate the presence of an impermeable boundary because, as noted above, the effect of such boundaries can be duplicated with an imaginary discharging well (1). Conversely, a recharge boundary causes drawdowns to be less than those defined by the trace of the type curve (3).



(3)



(4)

In the Jacob method, drawdowns begin to plot along a straight line after the test has been underway for some time (2) (4). The time at which the straight-line plot begins depends on the values of *T* and *S* of the aquifer and on the square of the distance between the observation well and the pumping well. (See "Time-Drawdown Analysis.") Values of T and S are determined from the first straight-line segment defined by the drawdowns after the start of the aquifer test. The slope of this straight line depends on the transmissivity (T) and on the pumping rate (Q). If a boundary is present, the drawdowns will depart from the first straight-line segment and begin to fall along another straight line (2) (4).

According to image-well theory, the effect of a recharge boundary can be duplicated by assuming that water is injected into the aquifer through a recharging image well at the same rate that water is being withdrawn from the real well. It follows, therefore, that, when the full effect of a recharge boundary is felt at an observation well, there will be no further increase in drawdown, and the water level in the well will stabilize. At this point in both the Theis and the Jacob methods, drawdowns plot along a straight line having a constant drawdown (3) (4). Conversely, an impermeable boundary causes the rate of drawdown to increase. In the Jacob method, as a result, the drawdowns plot along a new straight line having twice the slope as the line drawn through the drawdowns that occurred before the effect of the boundary was felt (2).

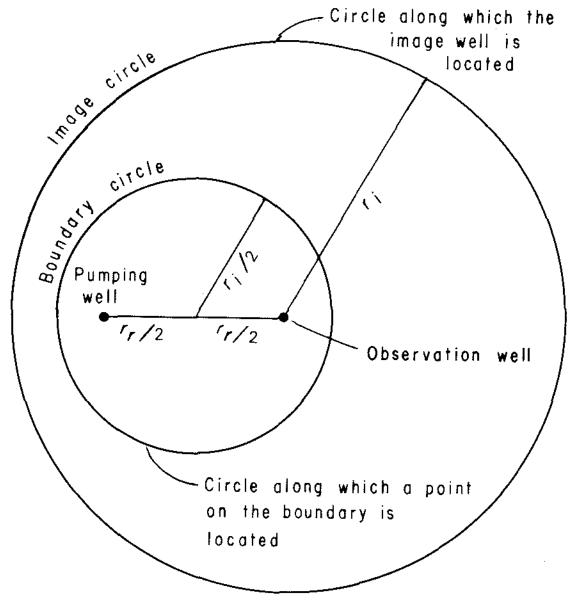
A word of caution should be injected here regarding use of the Jacob method when it is suspected that an aquifer test may be affected by boundary conditions. In many cases, the boundary begins to affect drawdowns before the method is applicable, the result being that T and S values determined from the data are erroneous, and the effect of the boundary is not identified. When it is suspected that an aquifer test may be affected by boundary conditions, the data should, at least initially, be analyzed with the Theis method.

The position and the nature of many boundaries are obvious. For example, the most common recharge boundaries are streams and lakes; possibly, the most common impermeable boundaries are the bedrock walls of alluvial valleys. The hydraulic distance to these boundaries, however, may not be obvious. A stream or lake may penetrate only a short distance into an aquifer, and their bottoms may be underlain by fine-grained material that hampers movement of water into the aquifer. Hydraulically, the boundaries formed by these surface-water bodies will appear to be farther from the pumping well than the near shore. Similarly, if a small amount of water moves across the bedrock wall of a valley, the hydraulic distance to the impermeable boundary will be greater than the distance to the valley wall.

Fortunately, the hydraulic distance to boundaries can be determined from the analysis of aquifer-test data. According to the Theis equation, if we deal with equal drawdowns caused by the real well and the image well (in other words, if $s_r = s_i$), then

$$\frac{r_r^2}{t_r} = \frac{r_i^2}{t_i} \tag{1}$$

where r_r is the distance from the observation well to the real well, r_i is the distance from the observation well to the image well, t_r is the time at which a drawdown of s_r is caused by the real well at the observation well, and t_i is the time at which a drawdown of s_i is caused by the image well at the observation well.



(5)

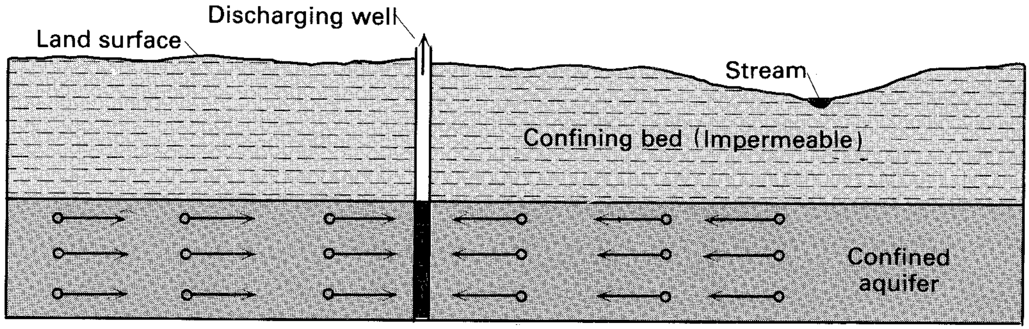
Solving equation 1 for the distance to the image well from the observation well, we obtain

$$r_i = r_r \sqrt{\frac{t_i}{t_r}} \tag{2}$$

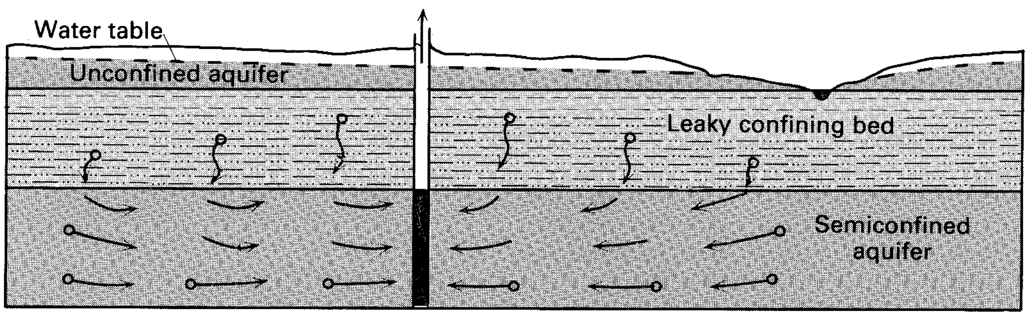
The image well is located at some point on a circle having a radius of r_i centered on the observation well (5). Because the image well is the same distance from the boundary as the real well, we know the boundary is halfway between the image well and the pumping well (5).

If the boundary is a stream or valley wall or some other feature whose physical position is obvious, its "hydraulic position" may be determined by using data from a single observation well. If, on the other hand, the boundary is the wall of a buried valley or some other feature not obvious from the land surface, distances to the image well from three observation wells may be needed to identify the position of the boundary.

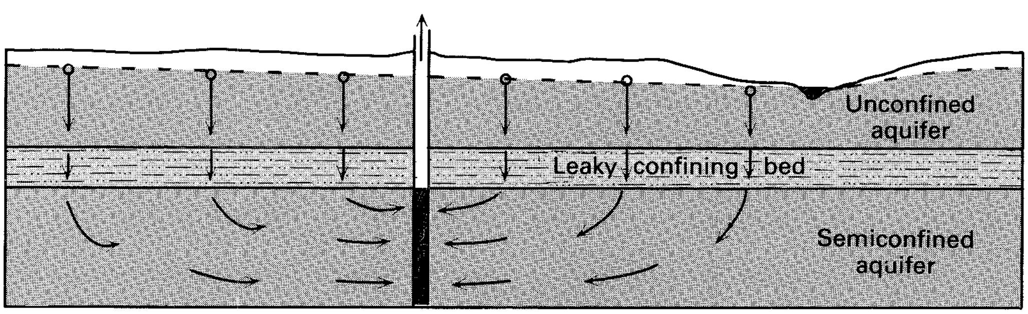
TESTS AFFECTED BY LEAKY CONFINING BEDS



(1)



(2)

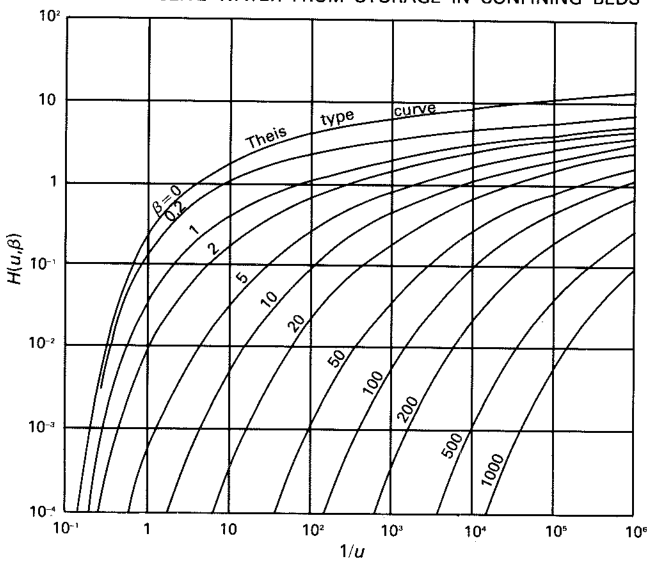


(3)

In the development of the Theis equation for the analysis of aquifer-test data, it was assumed that all water discharged from the pumping well was derived instantaneously from storage in the aquifer. (See "Analysis of Aquifer-Test Data.") Therefore, in the case of a confined aquifer, at least during the period of the test, the movement of water into the aquifer across its overlying and underlying confining beds is negligible. This assumption is satisfied by many confined aquifers. Many other aquifers, however, are bounded by leaky confining beds that transmit water into the aquifer in response to the withdrawals and cause drawdowns to differ from those that would be predicted by the Theis equation. The analysis of aquifer tests conducted on these aquifers requires the use of the methods that have been developed for semiconfined aquifers (also referred to in ground-water literature as "leaky aquifers").

Sketches 1 through 3 illustrate three different conditions commonly encountered in the field. Sketch 1 shows a confined aquifer bounded by thick, impermeable confining beds. Water initially pumped from such an aquifer is from storage, and aquifer-test data can be analyzed by using the Theis equation. Sketch 2 shows an aquifer overlain by a thick, leaky confining bed that, during an aquifer test, yields significant water from storage. The aquifer in this case may properly be referred to as a semiconfined aquifer, and the release of water from storage in the confining bed affects the analysis of aquifer-test data. Sketch 3 shows an aquifer overlain by a thin confining bed that does not yield significant water from storage but that

HANTUSH TYPE CURVES FOR SEMICONFINED AQUIFERS THAT RECEIVE WATER FROM STORAGE IN CONFINING BEDS



(4)

is sufficiently permeable to transmit water from the overlying unconfined aquifer into the semiconfined aquifer. Methods have been devised, largely by Madhi Hantush and C. E. Jacob, for use in analyzing the leaky conditions illustrated in sketches 2 and 3.

The use of these methods involves matching data plots with type curves, as the Theis method does. The major difference is that, whereas the Theis method involves use of a single type curve, the methods applicable to semiconfined aquifers involve "families" of type curves, each curve of which reflects different combinations of the hydraulic characteristics of the aquifer and the confining beds. Data plots of s versus t on logarithmic graph paper for aquifer tests affected by release of water from storage in the confining beds are matched to the family of type curves illustrated in sketch 4. For convenience, these curves are referred to as Hantush type. Four match-point coordinates are selected and substituted into the following equations to determine values of T and S:

$$T = \frac{QH(u,\beta)}{4\pi s} \quad (1)$$

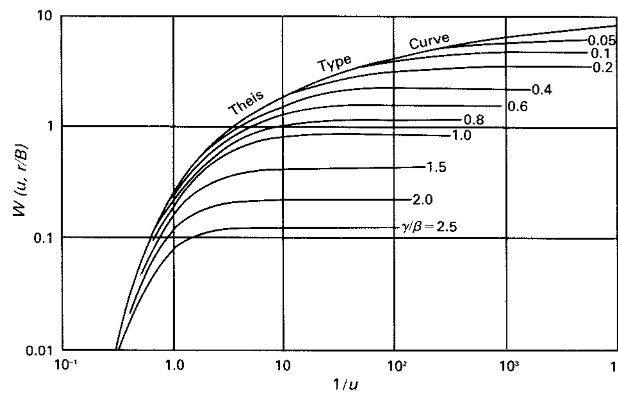
$$S = \frac{4Ttu}{r^2} \quad (2)$$

Data plots of s versus t on logarithmic graph paper for aquifer tests affected by leakage of water across confining beds are matched to the family of type curves shown in sketch 5. These type curves are based on equations developed by Hantush and Jacob and, for convenience, will be referred to as the Hantush-Jacob curves. The four coordinates of the match point are substituted into the following equations to determine T and S:

$$T = \frac{QW(u,r/B)}{4\pi s} \quad (3)$$

$$S = \frac{4Ttu}{r^2} \quad (4)$$

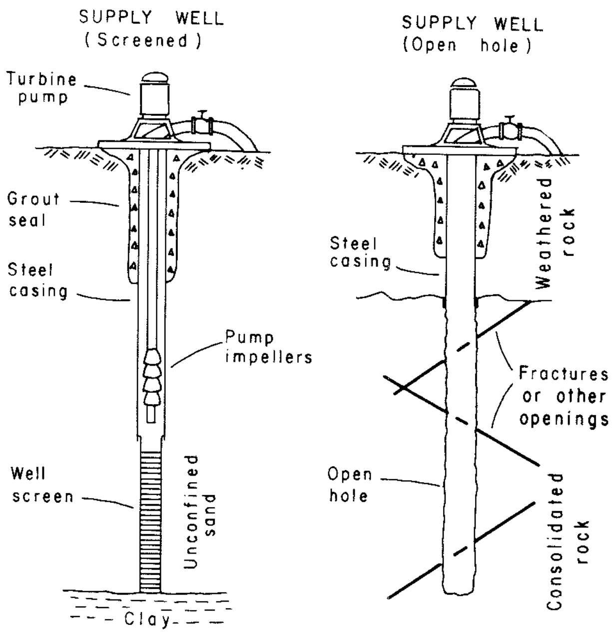
HANTUSH-JACOB TYPE CURVES FOR AQUIFERS RECEIVING LEAKAGE ACROSS CONFINING BEDS



(5)

In planning and conducting aquifer tests, hydrologists must give careful consideration to the hydraulic characteristics of the aquifer and to the type of boundary conditions (either recharge or impermeable) that are likely to exist in the vicinity of the test site. Following completion of the test, the next problem is to select the method of analysis that most closely represents the geologic and hydrologic conditions in the area affected by the test. When these conditions are not well known, the common practice is to prepare a data plot of s versus t on logarithmic paper and match it with the Theis type curve. If the data closely match the type curve, the values of T and S determined by using the Theis equation should be reliable. Significant departures of the data from the type curve generally reflect the presence of lateral boundaries or leaky confining beds. Both the geology of the area and the shape of the data plot may provide clues as to which of these conditions most likely exist. It is important to note, however, that some data plots for tests affected by impermeable boundaries are similar in shape to the Hantush curves.

WELL-CONSTRUCTION METHODS



(1) (2)

The seven different methods of well construction in fairly common use are listed in the table. The first four methods are limited to relatively shallow depths and are most commonly employed in the construction of domestic wells. One of the last three methods is usually employed in the construction of municipal and industrial wells and domestic wells in consolidated rock.

The objectives of well construction are to excavate a hole, usually of small diameter in comparison with the depth, to an aquifer and to provide a means for water to enter the hole while rock material is excluded. The means of excavating the hole is different for different methods.

Dug wells constructed with a pickax and shovel were relatively common in rural areas of the eastern and central parts of the country before the 1940's. Such wells are reasonably effective in fine-grained materials, such as glacial till, and thinly bedded sand and clay. The large irrigation ponds that extend below the water table, now being dug by bulldozer or dragline in the Atlantic Coastal Plain, are the modern version of the dug well.

Bored wells are constructed with earth augers turned either by hand or by power equipment and are the modern equivalent of the "hand-dug" well. Bored wells are relatively effective in material of low hydraulic conductivity and in areas underlain by thin surficial layers of silty and clayey sand.

Driven wells are constructed by driving a casing equipped with a screened drive point. Because of their relatively small diameter, these wells are suitable only for relatively permeable surficial aquifers. They are widely used as sources of domestic- and farm-water supplies in those parts of the Atlantic and Gulf Coastal Plains underlain by permeable sand.

Jetted wells are constructed by excavating a hole with a high-pressure jet of water. In dense clays, shell beds, and partially cemented layers, it may be necessary to attach a chisel bit to the jet pipe and alternately raise and drop the pipe to cut a hole.

The *percussion drilling method* (commonly referred to as the cable-tool method) consists of alternately raising and dropping a heavy weight equipped with a chisel bit. The rock at the bottom of the hole is thus shattered and, together with water, forms a slurry that is removed with a bailer. In unconsolidated material, the casing is driven a few feet at a time ahead of the drilling. After drilling to the maximum depth to be reached by the well, a screen is "telescoped" inside the casing and held in place while the casing is pulled back to expose the screen (1). The top of the screen is sealed against the casing by expanding a lead packer. In wells in consolidated

SUITABILITY OF DIFFERENT WELL-CONSTRUCTION METHODS TO GEOLOGIC CONDITIONS

[Modified from U.S. Environmental Protection Agency (1974), table 3]

					Drilled		
					Percussion	Rotary	
Characteristics	Dug	Bored	Driven	Jetted	(cable tool)	Hydraulic	Air
Maximum practical depth, in m (ft)	15 (50)	30 (100)	15 (50)	30 (100)	300 (1,000)	300 (1,000)	250 (800)
Range in diameter, in cm (in.)	1-6 m (3-20 ft)	5-75 (2-30)	3-6 (1-2)	5-30 (2-12)	10-46 (4-18)	10-61 (4-24)	10-25 (4-10)
Unconsolidated material:							
Silt	X	X	X	X	X	X	
Sand	X	X	X	X	X	X	
Gravel	X	X			X	X	
Glacial till	X	X			X	X	
Shell and limestone	X	X		X	X	X	
Consolidated material:							
Cemented gravel	X				X	X	X
Sandstone					X	X	X
Limestone					X	X	X
Shale					X	X	X
Igneous and metamorphic rocks					X	X	X

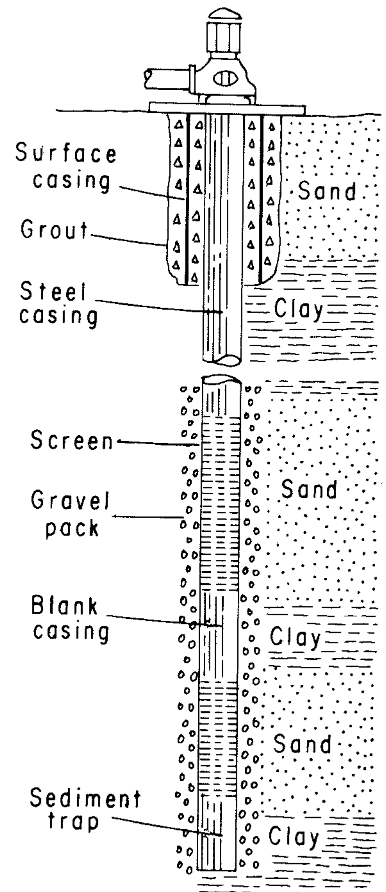
rock, the normal practice is to "seat" the casing firmly in the top of the rock and drill an open hole to the depth required to obtain the needed yield (2).

The hydraulic rotary method excavates a hole by rotating a drill pipe to which one of several types of drag or roller bits is attached. Water containing clay is circulated down the drill pipe in the "normal rotary" method and up the annular space, both to cool the bit and to remove the rock cuttings. In the "reverse rotary" method, the drilling fluid is circulated down the annular space and up the drill pipe. Clay in the drilling fluid adheres to the side of the hole and, together with the pressure exerted in the hole by the drilling fluid, prevents caving of the formation material. Thus, in the hydraulic rotary method, it is not necessary to install permanent-well casing during the drilling process. When the hole reaches the desired depth, a line of casing containing sections of screen at the desired intervals is lowered into the well. Hydraulic rotary is the method most commonly employed in drilling large-yield wells in areas underlain by thick sequences of unconsolidated deposits, such as the Atlantic and Gulf Coastal Plains. Where aquifers consist of alternating thin beds of sand and clay, the common practice is to install a gravel envelope around the screens. Such wells are referred to as *gravel packed* (3).

The *air rotary method* is similar to the hydraulic rotary method, except that the drilling fluid is air rather than mud. The air rotary method is suitable only for drilling in consolidated rocks. Most air rotary rigs are also equipped with mud pumps, which permit them to be used in the hydraulic rotary mode for drilling through saturated unconsolidated rock. This method is widely used in the construction of wells in fractured bedrock.

When the construction phase has been completed, it is necessary to begin the phase referred to as *well development*. The objective of this phase is to remove clay, silt, and fine-grained sand from the area adjacent to the screen or open hole so that the well will produce sediment-free water. The simplest method of development is to pump water from the well at a gradually increasing rate, the final rate being larger than the planned production rate. However, this method is not normally successful in screened and gravel-packed wells drilled by the hydraulic rotary method. For these wells, it is necessary to use a surge block or some other means to alternately force water into the formation and pull it back into the well. One of the most effective methods is to pump water under high pres-

SUPPLY WELL
(Multiple screen, gravel pack)



(3)

sure through orifices directed at the inside of the screen. The coarser grained particles pulled into the well during development tend to settle to the bottom of the well and must be removed with a bailer or pump. Chemicals that disperse clays and other fine-grained particles are also used as an aid in well development.